CHAPTER 11

PROBLEM 11.1

The motion of a particle is defined by the relation $x = 1.5t^4 - 30t^2 + 5t + 10$, where x and t are expressed in meters and seconds, respectively. Determine the position, the velocity, and the acceleration of the particle when $t = 4$ s.

SOLUTION

Given:

$$x = 1.5t^4 - 30t^2 + 5t + 10$$

$$v = \frac{dx}{dt} = 6t^3 - 60t + 5$$

$$a = \frac{dv}{dt} = 18t^2 - 60$$

Evaluate expressions at $t = 4$ s.

$$x = 1.5(4)^4 - 30(4)^2 + 5(4) + 10 = -66 \text{ m} \qquad x = -66.0 \text{ m} \blacktriangleleft$$

$$v = 6(4)^3 - 60(4) + 5 = 149 \text{ m/s} \qquad v = 149.0 \text{ m/s} \blacktriangleleft$$

$$a = 18(4)^2 - 60 = 228 \text{ m/s}^2 \qquad a = 228.0 \text{ m/s}^2 \blacktriangleleft$$

PROBLEM 11.2

The motion of a particle is defined by the relation $x = 12t^3 - 18t^2 + 2t + 5$, where x and t are expressed in meters and seconds, respectively. Determine the position and the velocity when the acceleration of the particle is equal to zero.

SOLUTION

Given:

$$x = 12t^3 - 18t^2 + 2t + 5$$

$$v = \frac{dx}{dt} = 36t^2 - 36t + 2$$

$$a = \frac{dv}{dt} = 72t - 36$$

Find the time for $a = 0$.

$$72t - 36 = 0 \Rightarrow t = 0.5 \text{ s}$$

Substitute into above expressions.

$$x = 12(0.5)^3 - 18(0.5)^2 + 2(0.5) + 5 = 3$$

$x = 3.00$ m ◀

$$v = 36(0.5)^2 - 36(0.5) + 2$$
$$= -7 \text{ m/s}$$

$v = -7.00$ m/s ◀

PROBLEM 11.3

The motion of a particle is defined by the relation $x = \frac{5}{3}t^3 - \frac{5}{2}t^2 - 30t + 8x$, where x and t are expressed in feet and seconds, respectively. Determine the time, the position, and the acceleration when $v = 0$.

SOLUTION

We have $$x = \frac{5}{3}t^3 - \frac{5}{2}t^2 - 30t + 8$$

Then $$v = \frac{dx}{dt} = 5t^2 - 5t - 30$$

and $$a = \frac{dv}{dt} = 10t - 5$$

When $v = 0$: $$5t^2 - 5t - 30 = 5(t^2 - t - 6) = 0$$

or $$t = 3 \text{ s} \quad \text{and} \quad t = -2 \text{ s} \quad \text{(Reject)}$$

$t = 3.00$ s ◀

At $t = 3$ s: $$x_3 = \frac{5}{3}(3)^3 - \frac{5}{2}(3)^2 - 30(3) + 8$$

or $x_3 = -59.5$ ft ◀

$$a_3 = 10(3) - 5$$

or $a_3 = 25.0 \text{ ft/s}^2$ ◀

PROBLEM 11.4

The motion of a particle is defined by the relation $x = 6t^2 - 8 + 40\cos \pi t$, where x and t are expressed in inches and seconds, respectively. Determine the position, the velocity, and the acceleration when $t = 6$ s.

SOLUTION

We have $x = 6t^2 - 8 + 40\cos \pi t$

Then $v = \dfrac{dx}{dt} = 12t - 40\pi \sin \pi t$

and $a = \dfrac{dv}{dt} = 12 - 40\pi^2 \cos \pi t$

At $t = 6$ s: $x_6 = 6(6)^2 - 8 + 40\cos 6\pi$ or $x_6 = 248$ in. ◀

$v_6 = 12(6) - 40\pi \sin 6\pi$ or $v_6 = 72.0$ in./s ◀

$a_6 = 12 - 40\pi^2 \cos 6\pi$ or $a_6 = -383$ in./s^2 ◀

PROBLEM 11.5

The motion of a particle is defined by the relation $x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$, where x and t are expressed in meters and seconds, respectively. Determine the time, the position, and the velocity when $a = 0$.

SOLUTION

We have $$x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$$

Then $$v = \frac{dx}{dt} = 24t^3 - 6t^2 - 24t + 3$$

and $$a = \frac{dv}{dt} = 72t^2 - 12t - 24$$

When $a = 0$: $$72t^2 - 12t - 24 = 12(6t^2 - t - 2) = 0$$

or $$(3t - 2)(2t + 1) = 0$$

or $$t = \frac{2}{3}\text{ s} \quad \text{and} \quad t = -\frac{1}{2}\text{ s} \quad \text{(Reject)}$$

$t = 0.667$ s ◀

At $t = \frac{2}{3}$ s: $$x_{2/3} = 6\left(\frac{2}{3}\right)^4 - 2\left(\frac{2}{3}\right)^3 - 12\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right) + 3$$

or $x_{2/3} = 0.259$ m ◀

$$v_{2/3} = 24\left(\frac{2}{3}\right)^3 - 6\left(\frac{2}{3}\right)^2 - 24\left(\frac{2}{3}\right) + 3$$

or $v_{2/3} = -8.56$ m/s ◀

PROBLEM 11.6

The motion of a particle is defined by the relation $x = 2t^3 - 15t^2 + 24t + 4$, where x is expressed in meters and t in seconds. Determine (a) when the velocity is zero, (b) the position and the total distance traveled when the acceleration is zero.

SOLUTION

$$x = 2t^3 - 15t^2 + 24t + 4$$

$$v = \frac{dx}{dt} = 6t^2 - 30t + 24$$

$$a = \frac{dv}{dt} = 12t - 30$$

(a) $\underline{v = 0 \text{ when}}$ $\quad 6t^2 - 30t + 24 = 0$

$$6(t-1)(t-4) = 0 \qquad t = 1.000 \text{ s} \quad \text{or} \quad t = 4.00 \text{ s} \quad ◀$$

(b) $\underline{a = 0 \text{ when}}$ $\quad 12t - 30 = 0 \qquad t = 2.5 \text{ s}$

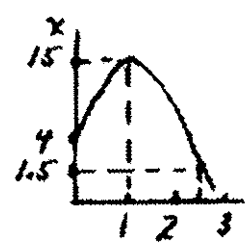
For $t = 2.5$ s: $\quad x_{2.5} = 2(2.5)^3 - 15(2.5)^2 + 24(2.5) + 4$

$$x_{2.5} = +1.500 \text{ m} \quad ◀$$

To find total distance traveled, we note that

$\underline{v = 0 \text{ when } t = 1 \text{ s}}$: $\quad x_1 = 2(1)^3 - 15(1)^2 + 24(1) + 4$

$$x_1 = +15 \text{ m}$$



For $t = 0$, $\quad x_0 = +4$ m

Distance traveled

From $t = 0$ to $t = 1$ s: $\quad x_1 - x_0 = 15 - 4 = 11 \text{ m} \rightarrow$

From $t = 1$ s to $t = 2.5$ s: $\quad x_{2.5} - x_1 = 1.5 - 15 = 13.5 \text{ m} \leftarrow$

Total distance traveled $= 11 \text{ m} + 13.5 \text{ m} = 24.5 \text{ m}$ ◀

PROBLEM 11.7

The motion of a particle is defined by the relation $x = t^3 - 6t^2 - 36t - 40$, where x and t are expressed in feet and seconds, respectively. Determine (a) when the velocity is zero, (b) the velocity, the acceleration, and the total distance traveled when $x = 0$.

SOLUTION

We have $$x = t^3 - 6t^2 - 36t - 40$$

Then $$v = \frac{dx}{dt} = 3t^2 - 12t - 36$$

and $$a = \frac{dv}{dt} = 6t - 12$$

(a) When $v = 0$: $$3t^2 - 12t - 36 = 3(t^2 - 4t - 12) = 0$$

or $$(t+2)(t-6) = 0$$

or $t = -2$ s (Reject) and $t = 6$ s $\qquad t = 6.00 \text{ s}$ ◀

(b) When $x = 0$: $$t^3 - 6t^2 - 36t - 40 = 0$$

Factoring $\quad (t-10)(t+2)(t+2) = 0 \quad$ or $\quad t = 10$ s

Now observe that $\quad 0 \le t < 6$ s: $\quad v < 0$

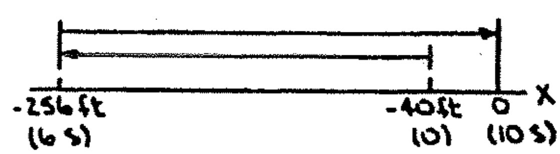
$6 \text{ s} < t \le 10$ s: $\quad v > 0$

and at $t = 0$: $\quad x_0 = -40$ ft

$t = 6$ s: $$x_6 = (6)^3 - 6(6)^2 - 36(6) - 40 = -256 \text{ ft}$$

$t = 10$ s: $$v_{10} = 3(10)^2 - 12(10) - 36 \qquad \text{or} \quad v_{10} = 144.0 \text{ ft/s}$$ ◀

$$a_{10} = 6(10) - 12 \qquad \text{or} \quad a_0 = 48.0 \text{ ft/s}^2$$ ◀



Then $$|x_6 - x_0| = |-256 - (-40)| = 216 \text{ ft}$$
$$x_{10} - x_6 = 0 - (-256) = 256 \text{ ft}$$

Total distance traveled $= (216 + 256)$ ft $= 472$ ft ◀

PROBLEM 11.8

The motion of a particle is defined by the relation $x = t^3 - 9t^2 + 24t - 8$, where x and t are expressed in inches and seconds, respectively. Determine (a) when the velocity is zero, (b) the position and the total distance traveled when the acceleration is zero.

SOLUTION

We have $$x = t^3 - 9t^2 + 24t - 8$$

Then $$v = \frac{dx}{dt} = 3t^2 - 18t + 24$$

and $$a = \frac{dv}{dt} = 6t - 18$$

(a) When $v = 0$: $$3t^2 - 18t + 24 = 3(t^2 - 6t + 8) = 0$$

or $$(t-2)(t-4) = 0$$

or $t = 2.00$ s and $t = 4.00$ s ◀

(b) When $a = 0$: $$6t - 18 = 0 \quad \text{or} \quad t = 3 \text{ s}$$

At $t = 3$ s: $$x_3 = (3)^3 - 9(3)^2 + 24(3) - 8$$ or $x_3 = 10.00$ in. ◀

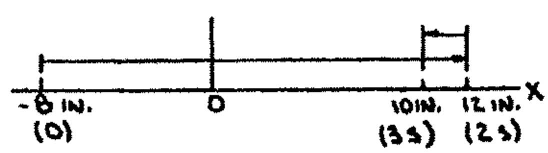
First observe that $0 \le t < 2$ s: $v > 0$

$2 \text{ s} < t \le 3$ s: $v < 0$

Now

At $t = 0$: $$x_0 = -8 \text{ in.}$$

At $t = 2$ s: $$x_2 = (2)^3 - 9(2)^2 + 24(2) - 8 = 12 \text{ in.}$$



Then $$x_2 - x_0 = 12 - (-8) = 20 \text{ in.}$$
$$|x_3 - x_2| = |10 - 12| = 2 \text{ in.}$$

Total distance traveled $= (20 + 2)$ in. $= 22.0$ in. ◀

PROBLEM 11.9

The acceleration of a particle is defined by the relation $a = -8\ \text{m/s}^2$. Knowing that $x = 20$ m when $t = 4$ s and that $x = 4$ m when $v = 16$ m/s, determine (*a*) the time when the velocity is zero, (*b*) the velocity and the total distance traveled when $t = 11$ s.

SOLUTION

We have $$\frac{dv}{dt} = a = -8\ \text{m/s}^2$$

Then $$\int dv = \int -8\,dt + C \quad C = \text{constant}$$

or $$v = -8t + C\ (\text{m/s})$$

Also $$\frac{dx}{dt} = v = -8t + C$$

At $t = 4$ s, $x = 20$ m: $$\int_{20}^{x} dx = \int_4^t (-8t + C)\,dt$$

or $$x - 20 = [-4t^2 + Ct]_4^t$$

or $$x = -4t^2 + C(t-4) + 84\ (\text{m})$$

When $v = 16$ m/s, $x = 4$ m: $$16 = -8t + C \Rightarrow C = 16 + 8t$$

$$4 = -4t^2 + C(t-4) + 84$$

Combining $$0 = -4t^2 + (16 + 8t)(t-4) + 80$$

Simplifying $$t^2 - 4t + 4 = 0$$

or $$t = 2\ \text{s}$$

and $$C = 32\ \text{m/s}$$

$$v = -8t + 32\ (\text{m/s})$$

$$x = -4t^2 + 32t - 44\ (\text{m})$$

(*a*) When $v = 0$: $-8t + 32 = 0$ or $t = 4.00$ s ◄

(*b*) Velocity and distance at 11 s.

$$v_{11} = -(8)(11) + 32$$ $v_{11} = -56.0$ m/s ◄

At $t = 0$: $x_0 = -44$ m

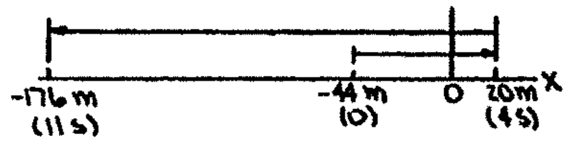
$t = 4$ s: $x_4 = 20$ m

$t = 11$ s: $x_{11} = -4(11)^2 + 32(11) - 44 = -176$ m

PROBLEM 11.9 (Continued)

Now observe that $0 \le t < 4 \text{ s}: \quad v > 0$

$$4 \text{ s} < t \le 11 \text{ s}: \quad v < 0$$



Then
$$x_4 - x_0 = 20 - (-44) = 64 \text{ m}$$
$$|x_{11} - x_4| = |-176 - 20| = 196 \text{ m}$$

Total distance traveled $= (64 + 196) \text{ m} = 260 \text{ m}$ ◀

PROBLEM 11.10

The acceleration of a particle is directly proportional to the square of the time t. When $t = 0$, the particle is at $x = 24$ m. Knowing that at $t = 6$ s, $x = 96$ m and $v = 18$ m/s, express x and v in terms of t.

SOLUTION

We have $$a = kt^2 \quad k = \text{constant}$$

Now $$\frac{dv}{dt} = a = kt^2$$

At $t = 6$ s, $v = 18$ m/s: $$\int_{18}^{v} dv = \int_{6}^{t} kt^2 dt$$

or $$v - 18 = \frac{1}{3}k(t^3 - 216)$$

or $$v = 18 + \frac{1}{3}k(t^3 - 216)(\text{m/s})$$

Also $$\frac{dx}{dt} = v = 18 + \frac{1}{3}k(t^3 - 216)$$

At $t = 0$, $x = 24$ m: $$\int_{24}^{x} dx = \int_{0}^{t}\left[18 + \frac{1}{3}k(t^3 - 216)\right]dt$$

or $$x - 24 = 18t + \frac{1}{3}k\left(\frac{1}{4}t^4 - 216t\right)$$

Now

At $t = 6$ s, $x = 96$ m: $$96 - 24 = 18(6) + \frac{1}{3}k\left[\frac{1}{4}(6)^4 - 216(6)\right]$$

or $$k = \frac{1}{9} \text{ m/s}^4$$

Then $$x - 24 = 18t + \frac{1}{3}\left(\frac{1}{9}\right)\left(\frac{1}{4}t^4 - 216t\right)$$

or $$x(t) = \frac{1}{108}t^4 + 10t + 24 \quad \blacktriangleleft$$

and $$v = 18 + \frac{1}{3}\left(\frac{1}{9}\right)(t^3 - 216)$$

or $$v(t) = \frac{1}{27}t^3 + 10 \quad \blacktriangleleft$$

PROBLEM 11.11

The acceleration of a particle is directly proportional to the time t. At $t=0$, the velocity of the particle is $v=16$ in./s. Knowing that $v=15$ in./s and that $x=20$ in. when $t=1$ s, determine the velocity, the position, and the total distance traveled when $t=7$ s.

SOLUTION

We have $$a = kt \quad k = \text{constant}$$

Now $$\frac{dv}{dt} = a = kt$$

At $t=0$, $v=16$ in./s: $$\int_{16}^{v} dv = \int_0^t kt\,dt$$

or $$v - 16 = \frac{1}{2}kt^2$$

or $$v = 16 + \frac{1}{2}kt^2 \text{(in./s)}$$

At $t=1$ s, $v=15$ in./s: $$15 \text{ in./s} = 16 \text{ in./s} + \frac{1}{2}k(1\text{ s})^2$$

or $$k = -2 \text{ in./s}^3 \quad \text{and} \quad v = 16 - t^2$$

Also $$\frac{dx}{dt} = v = 16 - t^2$$

At $t=1$ s, $x=20$ in.: $$\int_{20}^{x} dx = \int_1^t (16 - t^2)dt$$

or $$x - 20 = \left[16t - \frac{1}{3}t^3\right]_1^t$$

or $$x = -\frac{1}{3}t^3 + 16t + \frac{13}{3} \text{(in.)}$$

Then

At $t=7$ s: $$v_7 = 16 - (7)^2$$ or $v_7 = -33.0$ in./s ◀

$$x_7 = -\frac{1}{3}(7)^3 + 16(7) + \frac{13}{3}$$ or $x_7 = 2.00$ in. ◀

When $v=0$: $$16 - t^2 = 0 \quad \text{or} \quad t = 4 \text{ s}$$

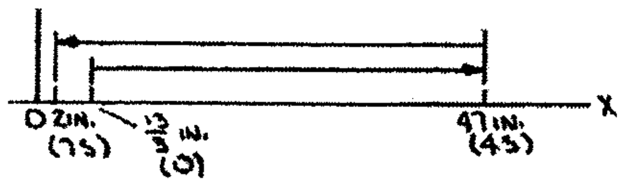
At $t = 0$: $x_0 = \dfrac{13}{3}$

$t = 4$ s: $x_4 = -\dfrac{1}{3}(4)^3 + 16(4) + \dfrac{13}{3} = 47$ in.

Now observe that

$0 \le t < 4$ s: $v > 0$

4 s $< t \le 7$ s: $v < 0$



Then
$$x_4 - x_0 = 47 - \frac{13}{3} = 42.67 \text{ in.}$$
$$|x_7 - x_4| = |2 - 47| = 45 \text{ in.}$$

Total distance traveled $= (42.67 + 45)$ in. $= 87.7$ in. ◀

PROBLEM 11.12

The acceleration of a particle is defined by the relation $a = kt^2$. (*a*) Knowing that $v = -32$ ft/s when $t = 0$ and that $v = +32$ ft/s when $t = 4$ s, determine the constant k. (*b*) Write the equations of motion, knowing also that $x = 0$ when $t = 4$ s.

SOLUTION

$$a = kt^2 \qquad (1)$$

$$\frac{dv}{dt} = a = kt^2$$

$t = 0,\ v = -32$ ft/s and $t = 4$ s, $v = +32$ ft/s

(*a*)

$$\int_{-32}^{32} dv = \int_0^4 kt^2\, dt$$

$$32 - (-32) = \frac{1}{3}k(4)^3 \qquad k = 3.00 \text{ ft/s}^4 \blacktriangleleft$$

(*b*) Substituting $k = 3$ ft/s^4 into (1)

$$\frac{dv}{dt} = a = 3t^2 \qquad a = 3t^2 \blacktriangleleft$$

$t = 0, v = -32$ ft/s:

$$\int_{-32}^{v} dv = \int_0^t 3t^2\, dt$$

$$v - (-32) = \frac{1}{3}3(t)^3 \qquad v = t^3 - 32 \blacktriangleleft$$

$$\frac{dx}{dt} = v = t^3 - 32$$

$t = 4$ s, $x = 0$:

$$\int_0^x dx = \int_4^t (t^3 - 32)dt; \quad x = \left|\frac{1}{4}t^4 - 32t\right|_4^t$$

$$x = \left[\frac{1}{4}t^4 - 32t\right] - \left[\frac{1}{4}(4)^4 - 32(4)\right]$$

$$x = \frac{1}{4}t^4 - 32t - 64 + 128 \qquad x = \frac{1}{4}t^4 - 32t + 64 \blacktriangleleft$$

PROBLEM 11.13

The acceleration of a particle is defined by the relation $a = A - 6t^2$, where A is a constant. At $t = 0$, the particle starts at $x = 8$ m with $v = 0$. Knowing that at $t = 1$ s, $v = 30$ m/s, determine (*a*) the times at which the velocity is zero, (*b*) the total distance traveled by the particle when $t = 5$ s.

SOLUTION

We have $\quad a = A - 6t^2 \quad A = \text{constant}$

Now $\quad \dfrac{dv}{dt} = a = A - 6t^2$

At $t = 0, v = 0$: $\quad \displaystyle\int_0^v dv = \int_0^t (A - 6t^2)dt$

or $\quad v = At - 2t^3 \text{ (m/s)}$

At $t = 1$ s, $v = 30$ m/s: $\quad 30 = A(1) - 2(1)^3$

or $\quad A = 32 \text{ m/s}^2 \quad \text{and} \quad v = 32t - 2t^3$

Also $\quad \dfrac{dx}{dt} = v = 32t - 2t^3$

At $t = 0, x = 8$ m: $\quad \displaystyle\int_8^x dx = \int_0^t (32t - 2t^3)dt$

or $\quad x = 8 + 16t^2 - \dfrac{1}{2}t^4 \text{ (m)}$

(*a*) When $v = 0$: $\quad 32t - 2t^3 = 2t(16 - t^2) = 0$

or $\quad t = 0 \quad \text{and} \quad t = 4.00 \text{ s} \quad ◀$

(*b*) At $t = 4$ s: $\quad x_4 = 8 + 16(4)^2 - \dfrac{1}{2}(4)^4 = 136 \text{ m}$

$t = 5$ s: $\quad x_5 = 8 + 16(5)^2 - \dfrac{1}{2}(5)^4 = 95.5 \text{ m}$

PROBLEM 11.13 (Continued)

Now observe that

$$0 < t < 4 \text{ s:} \qquad v > 0$$

$$4 \text{ s} < t \leq 5 \text{ s:} \qquad v < 0$$

Then

$$x_4 - x_0 = 136 - 8 = 128 \text{ m}$$

$$|x_5 - x_4| = |95.5 - 136| = 40.5 \text{ m}$$

Total distance traveled $= (128 + 40.5) \text{ m} = 168.5 \text{ m}$ ◀

PROBLEM 11.14

It is known that from $t = 2$ s to $t = 10$ s the acceleration of a particle is inversely proportional to the cube of the time t. When $t = 2$ s, $v = -15$ m/s, and when $t = 10$ s, $v = 0.36$ m/s. Knowing that the particle is twice as far from the origin when $t = 2$ s as it is when $t = 10$ s, determine (*a*) the position of the particle when $t = 2$ s, and when $t = 10$ s, (*b*) the total distance traveled by the particle from $t = 2$ s to $t = 10$ s.

SOLUTION

We have $$a = \frac{k}{t^3} \qquad k = \text{constant}$$

Now $$\frac{dv}{dt} = a = \frac{k}{t^3}$$

At $t = 2$ s, $v = -15$ m/s: $$\int_{-15}^{v} dv = \int_{2}^{t} \frac{k}{t^3}\,dt$$

or $$v - (-15) = -\frac{k^2}{2}\left[\frac{1}{t^2} - \frac{1}{(2)^2}\right]$$

or $$v = \frac{k}{2}\left(\frac{1}{4} - \frac{1}{t^2}\right) - 15 \text{ (m/s)}$$

At $t = 10$ s, $v = 0.36$ m/s: $$0.36 = \frac{k}{2}\left(\frac{1}{4} - \frac{1}{10^2}\right) - 15$$

or $$k = 128 \text{ m}\cdot\text{s}$$

and $$v = 1 - \frac{64}{t^2} \text{ (m/s)}$$

(*a*) We have $$\frac{dx}{dt} = v = 1 - \frac{64}{t^2}$$

Then $$\int dx = \int\left(1 - \frac{64}{t^2}\right)dt + C \qquad C = \text{constant}$$

or $$x = t + \frac{64}{t} + C \text{ (m)}$$

Now $x_2 = 2x_{10}$: $$2 + \frac{64}{2} + C = 2\left(10 + \frac{64}{10} + C\right)$$

or $$C = 1.2 \text{ m}$$

and $$x = t + \frac{64}{t} + 1.2 \text{ (m)}$$

PROBLEM 11.14 (Continued)

At $t = 2$ s: $\quad x_2 = 2 + \frac{64}{2} + 1.2 \quad$ or $\quad x_2 = 35.2 \text{ m}$ ◀

$t = 10$ s: $\quad x_{10} = 10 + \frac{64}{10} + 1.2 \quad$ or $\quad x_{10} = 17.60 \text{ m}$ ◀

Note: A second solution exists for the case $x_2 > 0$, $x_{10} < 0$. For this case, $C = -22\frac{4}{15}$ m

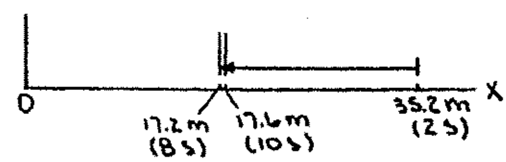
and $\quad x_2 = 11\frac{11}{15}$ m, $\quad x_{10} = -5\frac{13}{15}$ m

(*b*) When $v = 0$: $\quad 1 - \frac{64}{t^2} = 0 \quad$ or $\quad t = 8$ s

At $t = 8$ s: $\quad x_8 = 8 + \frac{64}{8} + 1.2 = 17.2$ m

Now observe that $2 \text{ s} \le t < 8 \text{ s}$: $\quad v < 0$

$8 \text{ s} < t \le 10 \text{ s}$: $\quad v > 0$



Then $\quad |x_8 - x_2| = |17.2 - 35.2| = 18$ m

$x_{10} - x_8 = 17.6 - 17.2 = 0.4$ m

Total distance traveled $= (18 + 0.4)$ m $= 18.40$ m ◀

Note: The total distance traveled is the same for both cases.

PROBLEM 11.15

The acceleration of a particle is defined by the relation $a = -k/x$. It has been experimentally determined that $v = 15$ ft/s when $x = 0.6$ ft and that $v = 9$ ft/s when $x = 1.2$ ft. Determine (*a*) the velocity of the particle when $x = 1.5$ ft, (*b*) the position of the particle at which its velocity is zero.

SOLUTION

$$a = \frac{vdv}{dx} = \frac{-k}{x}$$

Separate and integrate using $x = 0.6$ ft, $v = 15$ ft/s.

$$\int_{15}^{v} vdv = -k\int_{0.6}^{x} \frac{dx}{x}$$

$$\frac{1}{2}v^2\Big|_{15}^{v} = -k\ln x\Big|_{0.6}^{x}$$

$$\frac{1}{2}v^2 - \frac{1}{2}(15)^2 = -k\ln\left(\frac{x}{0.6}\right) \qquad (1)$$

When $v = 9$ ft/s, $x = 1.2$ ft

$$\frac{1}{2}(9)^2 - \frac{1}{2}(15)^2 = -k\ln\left(\frac{1.2}{0.6}\right)$$

Solve for k.

$$k = 103.874\ \text{ft}^2/\text{s}^2$$

(*a*) Substitute $k = 103.874\ \text{ft}^2/\text{s}^2$ and $x = 1.5$ ft into (1).

$$\frac{1}{2}v^2 - \frac{1}{2}(15)^2 = -103.874\ln\left(\frac{1.5}{0.6}\right)$$

$v = 5.89$ ft/s ◀

(*b*) For $v = 0$,

$$0 - \frac{1}{2}(15)^2 = -103.874\ln\left(\frac{x}{0.6}\right)$$

$$\ln\left(\frac{x}{0.6}\right) = 1.083$$

$x = 1.772$ ft ◀

PROBLEM 11.16

A particle starting from rest at $x = 1$ ft is accelerated so that its velocity doubles in magnitude between $x = 2$ ft and $x = 8$ ft. Knowing that the acceleration of the particle is defined by the relation $a = k[x - (A/x)]$, determine the values of the constants A and k if the particle has a velocity of 29 ft/s when $x = 16$ ft.

SOLUTION

We have $$v\frac{dv}{dx} = a = k\left(x - \frac{A}{x}\right)$$

When $x = 1$ ft, $v = 0$: $$\int_0^v v\,dv = \int_1^x k\left(x - \frac{A}{x}\right)dx$$

or $$\frac{1}{2}v^2 = k\left[\frac{1}{2}x^2 - A\ln x\right]_1^x$$

$$= k\left(\frac{1}{2}x^2 - A\ln x - \frac{1}{2}\right)$$

At $x = 2$ ft: $$\frac{1}{2}v_2^2 = k\left[\frac{1}{2}(2)^2 - A\ln 2 - \frac{1}{2}\right] = k\left(\frac{3}{2} - A\ln 2\right)$$

$x = 8$ ft: $$\frac{1}{2}v_8^2 = k\left[\frac{1}{2}(8)^2 - A\ln 8 - \frac{1}{2}\right] = k(31.5 - A\ln 8)$$

Now $\dfrac{v_8}{v_2} = 2$: $$\frac{\frac{1}{2}v_8^2}{\frac{1}{2}v_2^2} = (2)^2 = \frac{k(31.5 - A\ln 8)}{k\left(\frac{3}{2} - A\ln 2\right)}$$

or $$6 - 4A\ln 2 = 31.5 - A\ln 8$$

or $$25.5 = A(\ln 8 - 4\ln 2) = A(\ln 8 - \ln 2^4) = A\ln\left(\frac{1}{2}\right)$$

or $$A = -36.8 \text{ ft}^2 \blacktriangleleft$$

When $x = 16$ ft, $v = 29$ ft/s: $$\frac{1}{2}(29)^2 = k\left[\frac{1}{2}(16)^2 - \frac{25.5}{\ln\left(\frac{1}{2}\right)}\ln(16) - \frac{1}{2}\right]$$

Noting that $$\ln(16) = 4\ln 2 \quad \text{and} \quad \ln\left(\frac{1}{2}\right) = -\ln(2)$$

We have $$841 = k\left[236 - \frac{\ln 25.5}{-\ln(2)} = 4\ln(2) - 1\right]$$

or $$k = 1.832 \text{ s}^{-2} \blacktriangleleft$$

PROBLEM 11.17

A particle oscillates between the points $x = 40$ mm and $x = 160$ mm with an acceleration $a = k(100 - x)$, where a and x are expressed in mm/s^2 and mm, respectively, and k is a constant. The velocity of the particle is 18 mm/s when $x = 100$ mm and is zero at both $x = 40$ mm and $x = 160$ mm. Determine (a) the value of k, (b) the velocity when $x = 120$ mm.

SOLUTION

(a) We have
$$v\frac{dv}{dx} = a = k(100 - x)$$

When $x = 40$ mm, $v = 0$:
$$\int_0^v v\,dv = \int_{40}^x k(100 - x)dx$$

or
$$\frac{1}{2}v^2 = k\left[100x - \frac{1}{2}x^2\right]_{40}^x$$

or
$$\frac{1}{2}v^2 = k\left(100x - \frac{1}{2}x^2 - 3200\right)$$

When $x = 100$ mm, $v = 18$ mm/s:
$$\frac{1}{2}(18)^2 = k\left[100(100) - \frac{1}{2}(100)^2 - 3200\right]$$

or
$$k = 0.0900 \text{ s}^{-2} \blacktriangleleft$$

(b) When $x = 120$ mm:
$$\frac{1}{2}v^2 = 0.09\left[100(120) - \frac{1}{2}(120)^2 - 3200\right] = 144$$

or
$$v = \pm 16.97 \text{ mm/s} \blacktriangleleft$$

PROBLEM 11.18

A particle starts from rest at the origin and is given an acceleration $a = k/(x+4)^2$, where a and x are expressed in mm/s^2 and m, respectively, and k is a constant. Knowing that the velocity of the particle is 4 m/s when $x = 8$ m, determine (*a*) the value of k, (*b*) the position of the particle when $v = 4.5$ m/s, (*c*) the maximum velocity of the particle.

SOLUTION

(*a*) We have
$$v\frac{dv}{dx} = a = \frac{k}{(x+4)^2}$$

When $x = 0$, $v = 0$:
$$\int_0^v v\,dv = \int_0^x \frac{k}{(x+4)^2}\,dx$$

or
$$\frac{1}{2}v^2 = -k\left(\frac{1}{x+4} - \frac{1}{4}\right)$$

When $x = 8$ m, $v = 4$ m/s:
$$\frac{1}{2}(4)^2 = -k\left(\frac{1}{8+4} - \frac{1}{4}\right)$$

or $k = 48\ \text{m}^3/\text{s}^2$ ◀

(*b*) When $v = 4.5$ m/s:
$$\frac{1}{2}(4.5)^2 = -48\left(\frac{1}{x+4} - \frac{1}{4}\right)$$

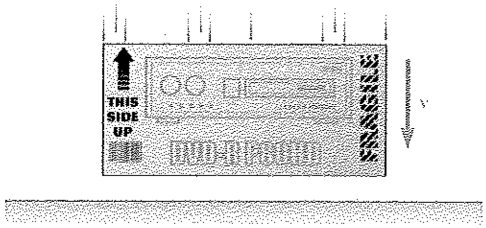
or $x = 21.6$ m ◀

(*c*) Note that when $v = v_{max}$, $a = 0$.

Now $a \to 0$ as $x \to \infty$ so that
$$\frac{1}{2}v_{max}^2 = 48 \lim_{x\to\infty}\left(\frac{1}{4} - \frac{1}{x+4}\right) = 48\left(\frac{1}{4}\right)$$

or $v_{max} = 4.90$ m/s ◀

PROBLEM 11.19



A piece of electronic equipment that is surrounded by packing material is dropped so that it hits the ground with a speed of 4 m/s. After impact the equipment experiences an acceleration of $a = -kx$, where k is a constant and x is the compression of the packing material. If the packing material experiences a maximum compression of 20 mm, determine the maximum acceleration of the equipment.

SOLUTION

$$a = \frac{v dv}{dx} = -kx$$

Separate and integrate.

$$\int_{v_0}^{v_f} v dv = -\int_0^{x_f} kx\, dx$$

$$\frac{1}{2}v_f^2 - \frac{1}{2}v_0^2 = -\frac{1}{2}kx^2\Big|_0^{x_f} = -\frac{1}{2}kx_f^2$$

Use $v_0 = 4$ m/s, $x_f = 0.02$ m, and $v_f = 0$. Solve for k.

$$0 - \frac{1}{2}(4)^2 = -\frac{1}{2}k(0.02)^2 \quad k = 40,000 \text{ s}^{-2}$$

Maximum acceleration.

$$a_{\max} = -kx_{\max}: \quad (-40,000)(0.02) = -800 \text{ m/s}^2$$

$a = 800 \text{ m/s}^2 \uparrow$ ◀

PROBLEM 11.20

Based on experimental observations, the acceleration of a particle is defined by the relation $a = -(0.1 + \sin x/b)$, where a and x are expressed in m/s^2 and meters, respectively. Knowing that $b = 0.8$ m and that $v = 1$ m/s when $x = 0$, determine (*a*) the velocity of the particle when $x = -1$ m, (*b*) the position where the velocity is maximum, (*c*) the maximum velocity.

SOLUTION

We have
$$v\frac{dv}{dx} = a = -\left(0.1 + \sin\frac{x}{0.8}\right)$$

When $x = 0$, $v = 1$ m/s:
$$\int_1^v v\,dv = \int_0^x -\left(0.1 + \sin\frac{x}{0.8}\right)dx$$

or
$$\frac{1}{2}(v^2 - 1) = -\left[0.1x - 0.8\cos\frac{x}{0.8}\right]_0^x$$

or
$$\frac{1}{2}v^2 = -0.1x + 0.8\cos\frac{x}{0.8} - 0.3$$

(*a*) When $x = -1$ m:
$$\frac{1}{2}v^2 = -0.1(-1) + 0.8\cos\frac{-1}{0.8} - 0.3$$

or $v = \pm 0.323$ m/s ◀

(*b*) When $v = v_{max}$, $a = 0$:
$$-\left(0.1 + \sin\frac{x}{0.8}\right) = 0$$

or $x = -0.080\,134$ m $x = -0.0801$ m ◀

(*c*) When $x = -0.080\,134$ m:
$$\frac{1}{2}v_{max}^2 = -0.1(-0.080\,134) + 0.8\cos\frac{-0.080\,134}{0.8} - 0.3$$
$$= 0.504\ \text{m}^2/\text{s}^2$$

or $v_{max} = 1.004$ m/s ◀

PROBLEM 11.21

Starting from $x = 0$ with no initial velocity, a particle is given an acceleration $a = 0.8\sqrt{v^2 + 49}$, where a and v are expressed in m/s^2 and m/s, respectively. Determine (a) the position of the particle when $v = 24$ m/s, (b) the speed of the particle when $x = 40$ m.

SOLUTION

We have
$$v\frac{dv}{dx} = a = 0.8\sqrt{v^2 + 49}$$

When $x = 0$, $v = 0$:
$$\int_0^v \frac{vdv}{\sqrt{v^2 + 49}} = \int_0^x 0.8\,dx$$

or
$$\left[\sqrt{v^2 + 49}\right]_0^v = 0.8x$$

or
$$\sqrt{v^2 + 49} - 7 = 0.8x$$

(a) When $v = 24$ m/s:
$$\sqrt{24^2 + 49} - 7 = 0.8x$$

or $x = 22.5$ m ◀

(b) When $x = 40$ m:
$$\sqrt{v^2 + 49} - 7 = 0.8(40)$$

or $v = 38.4$ m/s ◀

PROBLEM 11.22

The acceleration of a particle is defined by the relation $a = -k\sqrt{v}$, where k is a constant. Knowing that $x = 0$ and $v = 81$ m/s at $t = 0$ and that $v = 36$ m/s when $x = 18$ m, determine (a) the velocity of the particle when $x = 20$ m, (b) the time required for the particle to come to rest.

SOLUTION

(a) We have $$v\frac{dv}{dx} = a = -k\sqrt{v}$$

so that $$\sqrt{v}\,dv = -k\,dx$$

When $x = 0, v = 81$ m/s: $$\int_{81}^{v} \sqrt{v}\,dv = \int_{0}^{x} -k\,dx$$

or $$\frac{2}{3}[v^{3/2}]_{81}^{v} = -kx$$

or $$\frac{2}{3}[v^{3/2} - 729] = -kx$$

When $x = 18$ m, $v = 36$ m/s: $$\frac{2}{3}(36^{3/2} - 729) = -k(18)$$

or $$k = 19\sqrt{\text{m/s}^2}$$

Finally

When $x = 20$ m: $$\frac{2}{3}(v^{3/2} - 729) = -19(20)$$

or $$v^{3/2} = 159$$

$v = 29.3$ m/s ◀

(b) We have $$\frac{dv}{dt} = a = -19\sqrt{v}$$

At $t = 0, v = 81$ m/s: $$\int_{81}^{v} \frac{dv}{\sqrt{v}} = \int_{0}^{t} -19\,dt$$

or $$2[\sqrt{v}]_{81}^{v} = -19t$$

or $$2(\sqrt{v} - 9) = -19t$$

When $v = 0$: $$2(-9) = -19t$$

or $t = 0.947$ s ◀

PROBLEM 11.23

The acceleration of a particle is defined by the relation $a = -0.8v$, where a is expressed in in./s^2 and v in in./s. Knowing that at $t = 0$ the velocity is 40 in./s, determine (*a*) the distance the particle will travel before coming to rest, (*b*) the time required for the particle to come to rest, (*c*) the time required for the particle to be reduced by 50 percent of its initial value.

SOLUTION

(*a*) $$a = \frac{vdv}{dx} = -0.8v \qquad dv = -0.8dx$$

Separate and integrate with $v = 40$ in./s when $x = 0$.

$$\int_{40}^{v} dv = -0.8\int_{0}^{x} dx$$

$$v - 40 = -0.8x$$

<u>*Distance traveled.*</u>

For $v = 0$, $$x = \frac{-40}{-0.8} \Rightarrow x = 50.0 \text{ in.}$$ ◀

(*b*) $$a = \frac{dv}{dt} = -0.8v$$

Separate. $$\int_{40}^{v} \frac{dv}{v} = -\int_{0}^{x} 0.8dt$$

$$\ln v - \ln 40 = -0.8t$$

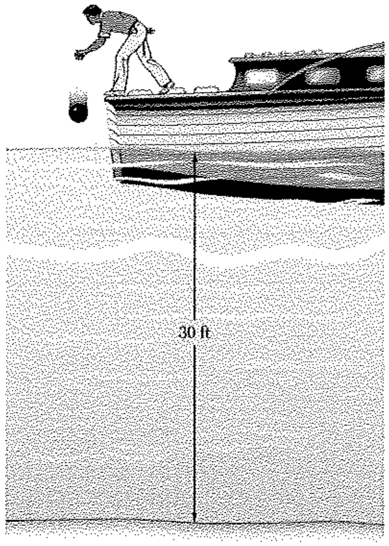
$$\ln\left(\frac{v}{40}\right) = -0.8t \qquad t = 1.25\ln\left(\frac{40}{v}\right)$$

For $v = 0$, we get $t = \infty$.

$t = \infty$ ◀

(*c*) For $v = 0.5(40 \text{ in./s}) = 20$ in./s,

$$t = 1.25\ln\left(\frac{40}{20}\right) = 0.866 \text{ s}$$ ◀



PROBLEM 11.24

A bowling ball is dropped from a boat so that it strikes the surface of a lake with a speed of 25 ft/s. Assuming the ball experiences a downward acceleration of $a = 10 - 0.9v^2$ when in the water, determine the velocity of the ball when it strikes the bottom of the lake.

SOLUTION

$$v_0 = 25 \text{ ft/s}, \quad x - x_0 = 30 \text{ ft}$$

$$a = 10 - 0.9v^2 = k(c^2 - v^2)$$

Where $\quad k = 0.9 \text{ ft}^{-1}$ and $c^2 = \dfrac{10}{0.9} = 11.111 \text{ ft}^2/\text{s}^2$

$$c = 3.3333 \text{ ft/s}$$

Since $v_0 > c$, write

$$a = v\frac{dv}{dx} = -k(v^2 - c^2)$$

$$\frac{vdv}{v^2 - c^2} = -kdx$$

Integrating,

$$\frac{1}{2}\ln(v^2 - c^2)\Big|_{v_0}^{v} = -k(x - x_0)$$

$$\ln\frac{v^2 - c^2}{v_0^2 - c^2} = -2k(x - x_0)$$

$$\frac{v^2 - c^2}{v_0^2 - c^2} = e^{-2k(x - x_0)}$$

$$v^2 = c^2 + \left(v_0^2 - c^2\right)e^{-2k(x-x_0)}$$

$$= 11.111 + [(25)^2 - 11.111]\,e^{-(2)(0.9)(30)}$$

$$= 11.111 + 3.89 \times 10^{-19} = 11.111 \text{ ft}^2/\text{s}^2$$

$v = 3.33$ ft/s ◀

PROBLEM 11.25

The acceleration of a particle is defined by the relation $a = 0.4(1 - kv)$, where k is a constant. Knowing that at $t = 0$ the particle starts from rest at $x = 4$ m and that when $t = 15$ s, $v = 4$ m/s, determine (a) the constant k, (b) the position of the particle when $v = 6$ m/s, (c) the maximum velocity of the particle.

SOLUTION

(a) We have $$\frac{dv}{dt} = a = 0.4(1 - kv)$$

At $t = 0$, $v = 0$: $$\int_0^v \frac{dv}{1 - kv} = \int_0^t 0.4 dt$$

or $$-\frac{1}{k}[\ln(1 - kv)]_0^v = 0.4t$$

or $$\ln(1 - kv) = -0.4kt \quad (1)$$

At $t = 15$ s, $v = 4$ m/s: $$\ln(1 - 4k) = -0.4k(15)$$
$$= -6k$$

Solving yields $$k = 0.145703 \text{ s/m}$$

or $k = 0.1457$ s/m ◀

(b) We have $$v\frac{dv}{dx} = a = 0.4(1 - kv)$$

When $x = 4$ m, $v = 0$: $$\int_0^v \frac{vdv}{1 - kv} = \int_4^x 0.4 dx$$

Now $$\frac{v}{1 - kv} = -\frac{1}{k} + \frac{1/k}{1 - kv}$$

Then $$\int_0^v \left[-\frac{1}{k} + \frac{1}{k(1 - kv)} \right] dv = \int_4^x 0.4 dx$$

or $$\left[-\frac{v}{k} - \frac{1}{k^2} \ln(1 - kv) \right]_0^v = 0.4[x]_4^x$$

or $$-\left[\frac{v}{k} + \frac{1}{k^2} \ln(1 - kv) \right] = 0.4(x - 4)$$

When $v = 6$ m/s: $$-\left[\frac{6}{0.145\,703} + \frac{1}{(0.145\,703)^2} \ln(1 - 0.145\,703 \times 6) \right] = 0.4(x - 4)$$

or $$0.4(x - 4) = 56.4778$$

or $x = 145.2$ m ◀

PROBLEM 11.25 (Continued)

(*c*) The maximum velocity occurs when $a = 0$.

$$a = 0: \quad 0.4(1 - kv_{\max}) = 0$$

or

$$v_{\max} = \frac{1}{0.145\ 703}$$

or

$$v_{\max} = 6.86 \text{ m/s} \blacktriangleleft$$

An alternative solution is to begin with Eq. (1).

$$\ln(1 - kv) = -0.4kt$$

Then

$$v = \frac{1}{k}(1 - k^{-0.4kt})$$

Thus, $v_{\max}$ is attained as $t \longrightarrow \infty$

$$v_{\max} = \frac{1}{k}$$

as above.

PROBLEM 11.26

A particle is projected to the right from the position $x = 0$ with an initial velocity of 9 m/s. If the acceleration of the particle is defined by the relation $a = -0.6v^{3/2}$, where a and v are expressed in m/s^2 and m/s, respectively, determine (*a*) the distance the particle will have traveled when its velocity is 4 m/s, (*b*) the time when $v = 1$ m/s, (*c*) the time required for the particle to travel 6 m.

SOLUTION

(*a*) We have
$$v\frac{dv}{dx} = a = -0.6v^{3/2}$$

When $x = 0$, $v = 9$ m/s:
$$\int_9^v v^{-(3/2)}dv = \int_0^x -0.6dx$$

or
$$2[v^{1/2}]_9^v = -0.6x$$

or
$$x = \frac{1}{0.3}(3 - v^{1/2}) \qquad (1)$$

When $v = 4$ m/s:
$$x = \frac{1}{0.3}(3 - 4^{1/2})$$

or $x = 3.33$ m ◀

(*b*) We have
$$\frac{dv}{dt} = a = -0.6v^{3/2}$$

When $t = 0$, $v = 9$ m/s:
$$\int_9^v v^{-(3/2)}dv = \int_0^t -0.6dt$$

or
$$-2[v^{-(1/2)}]_9^v = -0.6t$$

or
$$\frac{1}{\sqrt{v}} - \frac{1}{3} = 0.3t$$

When $v = 1$ m/s:
$$\frac{1}{\sqrt{1}} - \frac{1}{3} = 0.3t$$

or $t = 2.22$ s ◀

(*c*) We have
$$\frac{1}{\sqrt{v}} - \frac{1}{3} = 0.3t$$

or
$$v = \left(\frac{3}{1+0.9t}\right)^2 = \frac{9}{(1+0.9t)^2}$$

Now
$$\frac{dx}{dt} = v = \frac{9}{(1+0.9t)^2}$$

PROBLEM 11.26 (Continued)

At $t = 0,\ x = 0$:
$$\int_0^x dx = \int_0^t \frac{9}{(1+0.9t)^2}\,dt$$

or
$$x = 9\left[-\frac{1}{0.9}\frac{1}{1+0.9t}\right]_0^t$$
$$= 10\left(1-\frac{1}{1+0.9t}\right)$$
$$= \frac{9t}{1+0.9t}$$

When $x = 6$ m:
$$6 = \frac{9t}{1+0.9t}$$

or $t = 1.667$ s ◀

An alternative solution is to begin with Eq. (1).
$$x = \frac{1}{0.3}(3 - v^{1/2})$$

Then
$$\frac{dx}{dt} = v = (3-0.3x)^2$$

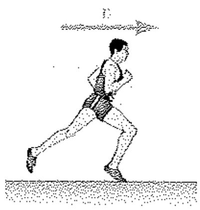
Now

At $t = 0,\ x = 0$:
$$\int_0^x \frac{dx}{(3-0.3x)^2} = \int_0^t dt$$

or
$$t = \frac{1}{0.3}\left[\frac{1}{3-0.3x}\right]_0^x = \frac{x}{9-0.9x}$$

Which leads to the same equation as above.

PROBLEM 11.27



Based on observations, the speed of a jogger can be approximated by the relation $v = 7.5(1-0.04x)^{0.3}$, where v and x are expressed in mi/h and miles, respectively. Knowing that $x = 0$ at $t = 0$, determine (*a*) the distance the jogger has run when $t = 1$ h, (*b*) the jogger's acceleration in ft/s^2 at $t = 0$, (*c*) the time required for the jogger to run 6 mi.

SOLUTION

(*a*) We have $$\frac{dx}{dt} = v = 7.5(1-0.04x)^{0.3}$$

At $t = 0$, $x = 0$: $$\int_0^x \frac{dx}{(1-0.04x)^{0.3}} = \int_0^t 7.5dt$$

or $$\frac{1}{0.7}\left(-\frac{1}{0.04}\right)[(1-0.04x)^{0.7}]_0^x = 7.5t$$

or $$1-(1-0.04x)^{0.7} = 0.21t \qquad (1)$$

or $$x = \frac{1}{0.04}[1-(1-0.21t)^{1/0.7}]$$

At $t = 1$ h: $$x = \frac{1}{0.04}\{1-[1-0.21(1)]^{1/0.7}\}$$

or $$x = 7.15 \text{ mi} ◀$$

(*b*) We have
$$\begin{aligned} a &= v\frac{dv}{dx} \\ &= 7.5(1-0.04x)^{0.3}\frac{d}{dx}[7.5(1-0.04x)^{0.3}] \\ &= 7.5^2(1-0.04x)^{0.3}[(0.3)(-0.04)(1-0.04x)^{-0.7}] \\ &= -0.675(1-0.04x)^{-0.4} \end{aligned}$$

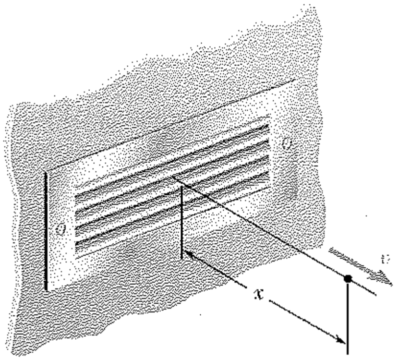
At $t = 0$, $x = 0$: $$a_0 = -0.675 \text{ mi/h}^2 \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2$$

or $$a_0 = -275\times10^{-6} \text{ ft/s}^2 ◀$$

(*c*) From Eq. (1) $$t = \frac{1}{0.21}[1-(1-0.04x)^{0.7}]$$

When $x = 6$ mi: $$\begin{aligned} t &= \frac{1}{0.21}\{1-[1-0.04(6)]^{0.7}\} \\ &= 0.83229 \text{ h} \end{aligned}$$

or $$t = 49.9 \text{ min} ◀$$



PROBLEM 11.28

Experimental data indicate that in a region downstream of a given louvered supply vent, the velocity of the emitted air is defined by $v = 0.18v_0/x$, where v and x are expressed in m/s and meters, respectively, and v_0 is the initial discharge velocity of the air. For $v_0 = 3.6$ m/s, determine (*a*) the acceleration of the air at $x = 2$ m, (*b*) the time required for the air to flow from $x = 1$ to $x = 3$ m.

SOLUTION

(*a*) We have

$$a = v\frac{dv}{dx}$$

$$= \frac{0.18v_0}{x}\frac{d}{dx}\left(\frac{0.18v_0}{x}\right)$$

$$= -\frac{0.0324v_0^2}{x^3}$$

When $x = 2$ m:

$$a = -\frac{0.0324(3.6)^2}{(2)^3}$$

or

$$a = -0.0525 \text{ m/s}^2 \blacktriangleleft$$

(*b*) We have

$$\frac{dx}{dt} = v = \frac{0.18v_0}{x}$$

From $x = 1$ m to $x = 3$ m:

$$\int_1^3 x\,dx = \int_{t_1}^{t_3} 0.18v_0\,dt$$

or

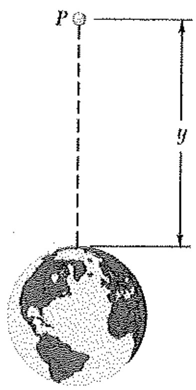
$$\left[\frac{1}{2}x^2\right]_1^3 = 0.18v_0(t_3 - t_1)$$

or

$$(t_3 - t_1) = \frac{\frac{1}{2}(9-1)}{0.18(3.6)}$$

or

$$t_3 - t_1 = 6.17 \text{ s} \blacktriangleleft$$



PROBLEM 11.29

The acceleration due to gravity at an altitude y above the surface of the earth can be expressed as

$$a = \frac{-32.2}{[1+(y/20.9\times10^6)]^2}$$

where a and y are expressed in ft/s^2 and feet, respectively. Using this expression, compute the height reached by a projectile fired vertically upward from the surface of the earth if its initial velocity is (*a*) 1800 ft/s, (*b*) 3000 ft/s, (*c*) 36,700 ft/s.

SOLUTION

We have

$$v\frac{dv}{dy} = a = -\frac{32.2}{\left(1+\frac{4}{20.9\times10^6}\right)^2}$$

When

$$y = 0, \qquad v = v_0$$

$$y = y_{\max}, \quad v = 0$$

Then

$$\int_{v_0}^{0} v\,dv = \int_0^{y_{\max}} \frac{-32.2}{\left(1+\frac{y}{20.9\times10^6}\right)^2}dy$$

or

$$-\frac{1}{2}v_0^2 = -32.2\left[-20.9\times10^6\frac{1}{1+\frac{4}{20.9\times10^6}}\right]_0^{y_{\max}}$$

or

$$v_0^2 = 1345.96\times10^6\left(1-\frac{1}{1+\frac{y_{\max}}{20.9\times10^6}}\right)$$

or

$$y_{\max} = \frac{v_0^2}{64.4-\frac{v_0^2}{20.9\times10^6}}$$

(*a*) $v_0 = 1800$ ft/s:

$$y_{\max} = \frac{(1800)^2}{64.4-\frac{(1800)^2}{20.9\times10^6}}$$

or $y_{\max} = 50.4\times10^3$ ft ◄

(*b*) $v_0 = 3000$ ft/s:

$$y_{\max} = \frac{(3000)^2}{64.4-\frac{(3000)^2}{20.9\times10^6}}$$

or $y_{\max} = 140.7\times10^3$ ft ◄

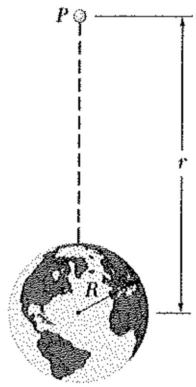
PROBLEM 11.29 (Continued)

(*c*) $v_0 = 36,700$ ft/s: $$y_{\max} = \frac{(36,700)^2}{64.4 - \frac{(36,700)^2}{20.9\times 10^6}}$$

or $$y_{\max} = -3.03\times 10^{10} \text{ ft} \blacktriangleleft$$

The velocity 36,700 ft/s is approximately the escape velocity v_R from the earth. For v_R

$$y_{\max} \to \infty \blacktriangleleft$$



PROBLEM 11.30

The acceleration due to gravity of a particle falling toward the earth is $a = -gR^2/r^2$, where r is the distance from the *center* of the earth to the particle, R is the radius of the earth, and g is the acceleration due to gravity at the surface of the earth. If $R = 3960$ mi, calculate the *escape velocity,* that is, the minimum velocity with which a particle must be projected vertically upward from the surface of the earth if it is not to return to the earth. (*Hint:* $v = 0$ for $r = \infty$.)

SOLUTION

We have
$$v\frac{dv}{dr} = a = -\frac{gR^2}{r^2}$$

When
$$r = R, \quad v = v_e$$
$$r = \infty, \quad v = 0$$

Then
$$\int_{v_e}^{0} v\,dv = \int_{R}^{\infty} -\frac{gR^2}{r^2}\,dr$$

or
$$-\frac{1}{2}v_e^2 = gR^2\left[\frac{1}{r}\right]_R^{\infty}$$

or
$$v_e = \sqrt{2gR}$$
$$= \left(2\times 32.2\ \text{ft/s}^2 \times 3960\ \text{mi} \times \frac{5280\ \text{ft}}{1\ \text{mi}}\right)^{1/2}$$

or
$$v_e = 36.7\times 10^3\ \text{ft/s} \blacktriangleleft$$

PROBLEM 11.31

The velocity of a particle is $v = v_0[1 - \sin(\pi t/T)]$. Knowing that the particle starts from the origin with an initial velocity v_0, determine (*a*) its position and its acceleration at $t = 3T$, (*b*) its average velocity during the interval $t = 0$ to $t = T$.

SOLUTION

(*a*) We have
$$\frac{dx}{dt} = v = v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]$$

At $t = 0, x = 0$:
$$\int_0^x dx = \int_0^t v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]dt$$

or
$$x = v_0\left[t + \frac{T}{\pi}\cos\left(\frac{\pi t}{T}\right)\right]_0^t$$

$$= v_0\left[t + \frac{T}{\pi}\cos\left(\frac{\pi t}{T}\right) - \frac{T}{\pi}\right] \qquad (1)$$

At $t = 3T$:
$$x_{BT} = v_0\left[3T + \frac{T}{\pi}\cos\left(\frac{\pi \times 3T}{T}\right) - \frac{T}{\pi}\right]$$
$$= v_0\left(3T - \frac{2T}{\pi}\right)$$

or
$$x_{3T} = 2.36\, v_0 T \blacktriangleleft$$

Also
$$a = \frac{dv}{dt} = \frac{d}{dt}\left\{v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]\right\} = -v_0\frac{\pi}{T}\cos\frac{\pi t}{T}$$

At $t = 3T$:
$$a_{3T} = -v_0\frac{\pi}{T}\cos\frac{\pi \times 3T}{T}$$

or
$$a_{3T} = \frac{\pi v_0}{T} \blacktriangleleft$$

(*b*) Using Eq. (1)

At $t = 0$:
$$x_0 = v_0\left[0 + \frac{T}{\pi}\cos(0) - \frac{T}{\pi}\right] = 0$$

PROBLEM 11.31 (Continued)

At $t=T$:

$$x_T = v_0\left[T+\frac{T}{\pi}\cos\left(\frac{\pi T}{T}\right)-\frac{T}{\pi}\right]$$

$$= v_0\left(T-\frac{2T}{\pi}\right)$$

$$= 0.363v_0T$$

Now

$$v_{\text{ave}} = \frac{x_T-x_0}{\Delta t} = \frac{0.363v_0T-0}{T-0}$$

or

$$v_{\text{ave}} = 0.363v_0 \blacktriangleleft$$

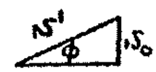
PROBLEM 11.32

The velocity of a slider is defined by the relation $v = v' \sin(\omega_n t + \phi)$. Denoting the velocity and the position of the slider at $t = 0$ by v_0 and x_0, respectively, and knowing that the maximum displacement of the slider is $2x_0$, show that (*a*) $v' = \left(v_0^2 + x_0^2\omega_n^2\right)/2x_0\omega_n$, (*b*) the maximum value of the velocity occurs when $x = x_0[3 - (v_0/x_0\omega_n)^2]/2$.

SOLUTION

(*a*) At $t = 0, v = v_0$: $$v_0 = v'\sin(0 + \phi) = v'\sin\phi$$

Then $$\cos\phi = \sqrt{v'^2 - v_0^2}\Big/v'$$



Now $$\frac{dx}{dt} = v = v'\sin(\omega_n t + \phi)$$

At $t = 0, x = x_0$: $$\int_{x0}^{x} dx = \int_0^t v'\sin(\omega_n t + \phi)dt$$

or $$x - x_0 = v'\left[-\frac{1}{\omega_n}\cos(\omega_n t + \phi)\right]_0^t$$

or $$x = x_0 + \frac{v'}{\omega_n}\left[\cos\phi - \cos(\omega_n t + \phi)\right]$$

Now observe that $x_{\max}$ occurs when $\cos(\omega_n t + \phi) = -1$. Then

$$x_{\max} = 2x_0 = x_0 + \frac{v'}{\omega_n}[\cos\phi - (-1)]$$

Substituting for $\cos\phi$ $$x_0 = \frac{v'}{\omega_n}\left(\frac{\sqrt{v'^2 - v_0^2}}{v^1} + 1\right)$$

or $$x_0\omega_n - v^1 = \sqrt{v'^2 - v_0^2}$$

Squaring both sides of this equation

$$x_0^2\omega_n^2 - 2x_0\omega_n + v'^2 = v'^2 - v_0^2$$

or $$v' = \frac{v_0^2 + x_0^2\omega_n^2}{2x_0\omega_n}$$ Q. E. D.

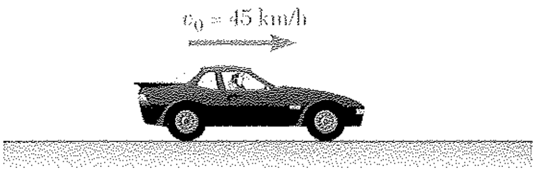
PROBLEM 11.32 (Continued)

(*b*) First observe that $v_{\max}$ occurs when $\omega_n t + \phi = \frac{\pi}{2}$. The corresponding value of x is

$$
\begin{aligned}
x_{v_{\max}} &= x_0 + \frac{v'}{\omega_n}\left[\cos\phi - \cos\left(\frac{\pi}{2}\right)\right] \\
&= x_0 + \frac{v'}{\omega_n}\cos\phi
\end{aligned}
$$

Substituting first for $\cos\phi$ and then for v'

$$
\begin{aligned}
x_{v_{\max}} &= x_0 + \frac{v'}{\omega_n}\frac{\sqrt{v'^2 - v_0^2}}{v'} \\
&= x_0 + \frac{1}{\omega_n}\left[\left(\frac{v_0^2 + x_0^2\omega_n^2}{2x_0\omega_n}\right)^2 - v_0^2\right]^{1/2} \\
&= x_0 + \frac{1}{2x_0\omega_n^2}\left(v_0^4 + 2v_0^2x_0^2\omega_n^2 + x_0^4\omega_n^4 - 4x_0^2\omega_n^2v_0^2\right)^{1/2} \\
&= x_0 + \frac{1}{2x_0\omega_n^2}\left[\left(x_0^2\omega_n^2 - v_0^2\right)^2\right]^{1/2} \\
&= x_0 + \frac{x_0^2\omega_n^2 - v_0^2}{2x_0\omega_n^2} \\
&= \frac{x_0}{2}\left[3 - \left(\frac{v_0}{x_0\omega_n}\right)^2\right] \qquad \text{Q. E. D.}
\end{aligned}
$$



PROBLEM 11.33

A motorist enters a freeway at 45 km/h and accelerates uniformly to 99 km/h. From the odometer in the car, the motorist knows that she traveled 0.2 km while accelerating. Determine (*a*) the acceleration of the car, (*b*) the time required to reach 99 km/h.

SOLUTION

(*a*) Acceleration of the car.

$$v_1^2 = v_0^2 + 2a(x_1 - x_0)$$

$$a = \frac{v_1^2 - v_0^2}{2(x_1 - x_0)}$$

Data:

$$v_0 = 45 \text{ km/h} = 12.5 \text{ m/s}$$
$$v_1 = 99 \text{ km/h} = 27.5 \text{ m/s}$$

$$x_0 = 0$$
$$x_1 = 0.2 \text{ km} = 200 \text{ m}$$

$$a = \frac{(27.5)^2 - (12.5)^2}{(2)(200 - 0)}$$

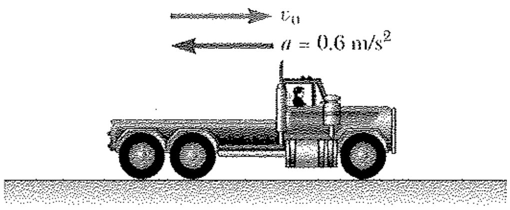
$a = 1.500 \text{ m/s}^2$ ◀

(*b*) Time to reach 99 km/h.

$$v_1 = v_0 + a(t_1 - t_0)$$

$$t_1 - t_0 = \frac{v_1 - v_0}{a}$$
$$= \frac{27.5 - 12.5}{1.500}$$
$$= 10.00 \text{ s}$$

$t_1 - t_0 = 10.00 \text{ s}$ ◀



PROBLEM 11.34

A truck travels 220 m in 10 s while being decelerated at a constant rate of 0.6 m/s^2. Determine (*a*) its initial velocity, (*b*) its final velocity, (*c*) the distance traveled during the first 1.5 s.

SOLUTION

(*a*) Initial velocity.

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v_0 = \frac{x - x_0}{t} - \frac{1}{2}at$$

$$= \frac{220}{10} - \frac{1}{2}(-0.6)(10)$$

$v_0 = 25.9$ m/s ◀

(*b*) Final velocity.

$$v = v_0 + at$$

$$v = 25.0 + (-0.6)(10)$$

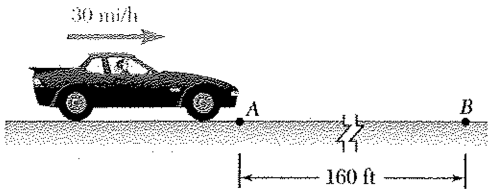
$v_f = 19.00$ m/s ◀

(*c*) Distance traveled during first 1.5 s.

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$= 0 + (25.0)(1.5) + \frac{1}{2}(-0.6)(1.5)^2$$

$x = 36.8$ m ◀



PROBLEM 11.35

Assuming a uniform acceleration of 11 ft/s^2 and knowing that the speed of a car as it passes A is 30 mi/h, determine (*a*) the time required for the car to reach B, (*b*) the speed of the car as it passes B.

SOLUTION

(*a*) Time required to reach B.

$$v_A = 30 \text{ mi/h} = 44 \text{ ft/s}, \quad x_A = 0, \quad x_B = 160 \text{ ft}, \quad a = 11 \text{ ft/s}^2$$

$$x_B = x_A + v_A t + \frac{1}{2}at^2$$

$$160 = 0 + 44t + \frac{1}{2}(11)t^2$$

$$5.5t^2 + 44t - 160 = 0$$

$$t = \frac{-44 \pm \sqrt{(44)^2 - (4)(5.5)(-160)}}{(2)(5.5)}$$

$$= -4 \pm 6.7150$$

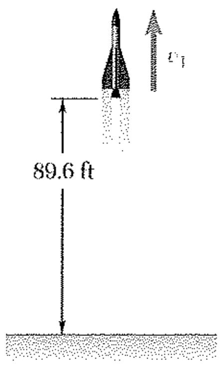
Rejecting the negative root. $t = 2.7150 \text{ s}$ $\qquad t = 2.71 \text{ s}$ ◀

(*b*) Speed at B.

$$v_B = v_A + at = 44 + (11)(2.7150) = 73.865 \text{ ft/s}$$

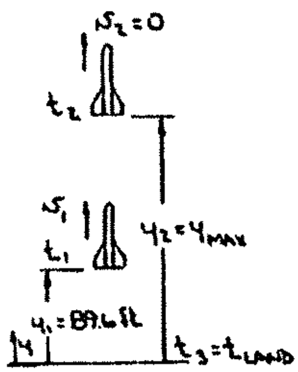
$v_B = 50.4 \text{ mi/h}$ ◀

PROBLEM 11.36



A group of students launches a model rocket in the vertical direction. Based on tracking data, they determine that the altitude of the rocket was 89.6 ft at the end of the powered portion of the flight and that the rocket landed 16 s later. Knowing that the descent parachute failed to deploy so that the rocket fell freely to the ground after reaching its maximum altitude and assuming that $g = 32.2\ \text{ft/s}^2$, determine (*a*) the speed v_1 of the rocket at the end of powered flight, (*b*) the maximum altitude reached by the rocket.

SOLUTION



(*a*) We have $\quad y = y_1 + v_1 t + \frac{1}{2}at^2$

At t_{land}, $\quad y = 0$

Then
$$0 = 89.6\ \text{ft} + v_1(16\ \text{s}) + \frac{1}{2}(-32.2\ \text{ft/s}^2)(16\ \text{s})^2$$

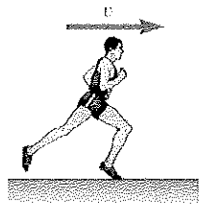
or $\quad v_1 = 2.52\ \text{ft/s}$ ◀

(*b*) We have $\quad v^2 = v_1^2 + 2a(y - y_1)$

At $\quad y = y_{\max}, \quad v = 0$

Then $\quad 0 = (252\ \text{ft/s})^2 + 2(-32.2\ \text{ft/s}^2)(y_{\max} - 89.6)\ \text{ft}$

or $\quad y_{\max} = 1076\ \text{ft}$ ◀



PROBLEM 11.37

A sprinter in a 100-m race accelerates uniformly for the first 35 m and then runs with constant velocity. If the sprinter's time for the first 35 m in 5.4 s, determine (*a*) his acceleration, (*b*) his final velocity, (*c*) his time for the race.

SOLUTION

<u>Given</u>: $0 \le x \le 35$ m, $a = \text{constant}$

$35 \text{ m} < x \le 100$ m, $v = \text{constant}$

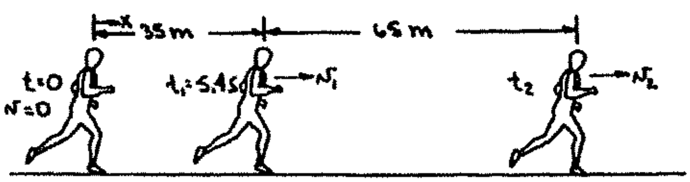
At $t = 0$, $v = 0$ when $x = 35$ m, $t = 5.4$ s

Find:

(*a*) a

(*b*) v when $x = 100$ m

(*c*) t when $x = 100$ m



(*a*) We have $\quad x = 0 + 0t + \frac{1}{2}at^2 \quad$ for $\quad 0 \le x \le 35$ m

At $t = 5.4$ s: $\quad 35 \text{ m} = \frac{1}{2}a(5.4 \text{ s})^2$

or $\quad a = 2.4005 \text{ m/s}^2$

$a = 2.40 \text{ m/s}^2$ ◀

(*b*) First note that $v = v_{\text{max}}$ for $35 \text{ m} \le x \le 100$ m.

Now $\quad v^2 = 0 + 2a(x - 0) \quad$ for $\quad 0 \le x \le 35$ m

When $x = 35$ m: $\quad v_{\text{max}}^2 = 2(2.4005 \text{ m/s}^2)(35 \text{ m})$

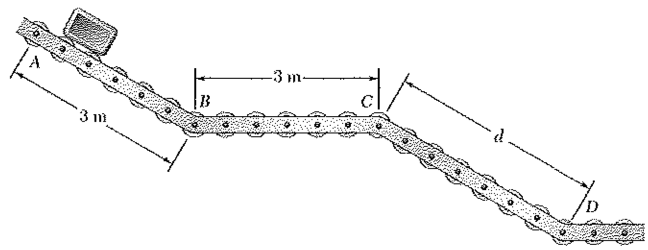
or $\quad v_{\text{max}} = 12.9628 \text{ m/s} \qquad v_{\text{max}} = 12.96 \text{ m/s}$ ◀

(*c*) We have $\quad x = x_1 + v_0(t - t_1) \quad$ for $\quad 35 \text{ m} < x \le 100$ m

When $x = 100$ m: $\quad 100 \text{ m} = 35 \text{ m} + (12.9628 \text{ m/s})(t_2 - 5.4) \text{ s}$

or $\qquad t_2 = 10.41$ s ◀

PROBLEM 11.38



A small package is released from rest at A and moves along the skate wheel conveyor $ABCD$. The package has a uniform acceleration of $4.8\ \text{m/s}^2$ as it moves down sections AB and CD, and its velocity is constant between B and C. If the velocity of the package at D is 7.2 m/s, determine (a) the distance d between C and D, (b) the time required for the package to reach D.

SOLUTION

(a) For $A \to B$

and $C \to D$

we have $$v^2 = v_0^2 + 2a(x - x_0)$$

Then,

at B $$v_{BC}^2 = 0 + 2(4.8\ \text{m/s}^2)(3-0)\ \text{m}$$

$$= 28.8\ \text{m}^2/\text{s}^2 \qquad (v_{BC} = 5.3666\ \text{m/s})$$

and at D $$v_D^2 = v_{BC}^2 + 2a_{CD}(x_D - x_C) \qquad d = x_d - x_C$$

or $$(7.2\ \text{m/s})^2 = (28.8\ \text{m}^2/\text{s}^2) + 2(4.8\ \text{m/s}^2)d$$

or $$d = 2.40\ \text{m} \blacktriangleleft$$

(b) For $A \to B$

and $C \to D$,

we have $$v = v_0 + at$$

Then $A \to B$

$$5.3666\ \text{m/s} = 0 + (4.8\ \text{m/s}^2)t_{AB}$$

or $$t_{AB} = 1.11804\ \text{s}$$

and $C \to D$ $$7.2\ \text{m/s} = 5.3666\ \text{m/s} + (4.8\ \text{m/s}^2)t_{CD}$$

or $$t_{CD} = 0.38196\ \text{s}$$

PROBLEM 11.38 (Continued)

Now,

for $B \to C$,

we have $x_C = x_B + v_{BC} t_{BC}$

or $3 \text{ m} = (5.3666 \text{ m/s}) t_{BC}$

or $t_{BC} = 0.55901 \text{ s}$

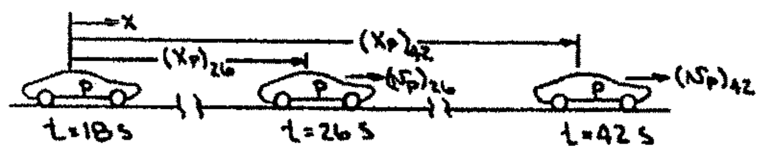
Finally, $t_D = t_{AB} + t_{BC} + t_{CD} = (1.11804 + 0.55901 + 0.38196) \text{ s}$

or $t_D = 2.06 \text{ s}$ ◀

PROBLEM 11.39

A police officer in a patrol car parked in a 70 km/h speed zone observes a passing automobile traveling at a slow, constant speed. Believing that the driver of the automobile might be intoxicated, the officer starts his car, accelerates uniformly to 90 km/h in 8 s, and, maintaining a constant velocity of 90 km/h, overtakes the motorist 42 s after the automobile passed him. Knowing that 18 s elapsed before the officer began pursuing the motorist, determine (*a*) the distance the officer traveled before overtaking the motorist, (*b*) the motorist's speed.

SOLUTION



$(v_P)_{18} = 0 \qquad (v_P)_{26} = 90 \text{ km/h} = 25 \text{ m/s} \qquad (v_P)_{42} = 90 \text{ km/h} = 25 \text{ m/s}$

(*a*) Patrol car:

For $18 \text{ s} < t \leq 26 \text{ s}$: $\quad v_P = 0 + a_P(t - 18)$

At $t = 26$ s: $\quad 25 \text{ m/s} = a_P(26 - 18) \text{ s}$

or $\quad a_P = 3.125 \text{ m/s}^2$

Also, $\quad x_P = 0 + 0(t - 18) - \dfrac{1}{2}a_P(t - 18)^2$

At $t = 26$ s: $\quad (x_P)_{26} = \dfrac{1}{2}(3.125 \text{ m/s}^2)(26 - 18)^2 = 100 \text{ m}$

For $26 \text{ s} < t \leq 42 \text{ s}$: $\quad x_P = (x_P)_{26} + (v_P)_{26}(t - 26)$

At $t = 42$ s: $\quad (x_P)_{42} = 100 \text{ m} + (25 \text{ m/s})(42 - 26)\text{s}$
$\quad = 500 \text{ m}$

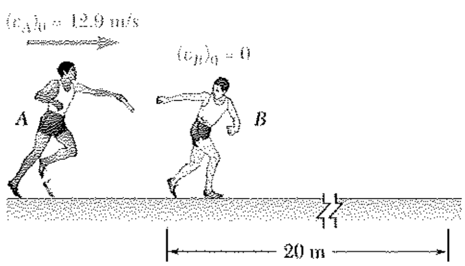
$(x_P)_{42} = 0.5 \text{ km}$ ◀

(*b*) For the motorist's car: $\quad x_M = 0 + v_M t$

At $t = 42$ s, $x_M = x_P$: $\quad 500 \text{ m} = v_M(42 \text{ s})$

or $\quad v_M = 11.9048 \text{ m/s}$

or $\quad v_M = 42.9 \text{ km/h}$ ◀



PROBLEM 11.40

As relay runner A enters the 20-m-long exchange zone with a speed of 12.9 m/s, he begins to slow down. He hands the baton to runner B 1.82 s later as they leave the exchange zone with the same velocity. Determine (a) the uniform acceleration of each of the runners, (b) when runner B should begin to run.

SOLUTION

(a) For runner A: $$x_A = 0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

At $t = 1.82$ s: $$20\text{ m} = (12.9\text{ m/s})(1.82\text{ s}) + \frac{1}{2} a_A (1.82\text{ s})^2$$

or $$a_A = -2.10\text{ m/s}^2 \blacktriangleleft$$

Also $$v_A = (v_A)_0 + a_A t$$

At $t = 1.82$ s: $$(v_A)_{1.82} = (12.9\text{ m/s}) + (-2.10\text{ m/s}^2)(1.82\text{ s})$$
$$= 9.078\text{ m/s}$$

For runner B: $$v_B^2 = 0 + 2a_B\left[x_B - 0\right]$$

When $$x_B = 20\text{ m},\quad v_B = v_A:\quad (9.078\text{ m/s})^2 + 2a_B(20\text{ m})$$

or $$a_B = 2.0603\text{ m/s}^2$$

$$a_B = 2.06\text{ m/s}^2 \blacktriangleleft$$

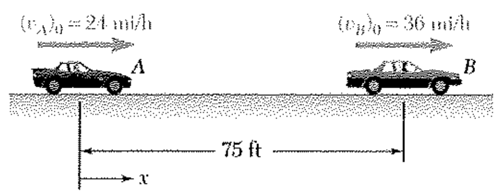
(b) For runner B: $$v_B = 0 + a_B(t - t_B)$$

where t_B is the time at which he begins to run.

At $t = 1.82$ s: $$9.078\text{ m/s} = (2.0603\text{ m/s}^2)(1.82 - t_B)\text{s}$$

or $$t_B = -2.59\text{ s}$$

Runner B should start to run 2.59 s before A reaches the exchange zone. ◀



PROBLEM 11.41

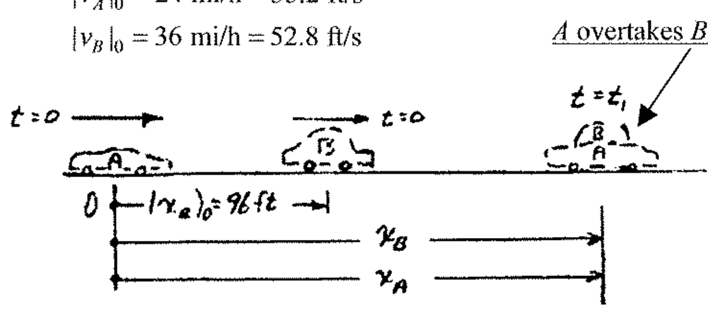
Automobiles A and B are traveling in adjacent highway lanes and at $t = 0$ have the positions and speeds shown. Knowing that automobile A has a constant acceleration of 1.8 ft/s^2 and that B has a constant deceleration of 1.2 ft/s^2, determine (a) when and where A will overtake B, (b) the speed of each automobile at that time.

SOLUTION

$$a_A = +1.8 \text{ ft/s}^2 \quad a_B = -1.2 \text{ ft/s}^2$$

$$|v_A|_0 = 24 \text{ mi/h} = 35.2 \text{ ft/s}$$

$$|v_B|_0 = 36 \text{ mi/h} = 52.8 \text{ ft/s}$$



Motion of auto A:

$$v_A = (v_A)_0 + a_A t = 35.2 + 1.8t \tag{1}$$

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2 = 0 + 35.2t + \frac{1}{2}(1.8)t^2 \tag{2}$$

Motion of auto B:

$$v_B = (v_B)_0 + a_B t = 52.8 - 1.2t \tag{3}$$

$$x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2 = 75 + 52.8t + \frac{1}{2}(-1.2)t^2 \tag{4}$$

(a) A overtakes B at $t = t_1$.

$$x_A = x_B: \quad 35.2t + 0.9t_1^2 = 75 + 52.8t_1 - 0.6t_1^2$$

$$1.5t_1^2 - 17.6t_1 - 75 = 0$$

$$t_1 = -3.22 \text{ s} \quad \text{and} \quad t_1 = 15.0546$$

$t_1 = 15.05$ s ◀

Eq. (2):

$$x_A = 35.2(15.05) + 0.9(15.05)^2$$

$x_A = .734$ ft ◀

PROBLEM 11.41 (Continued)

(*b*) Velocities when $t_1 = 15.05 \text{ s}$

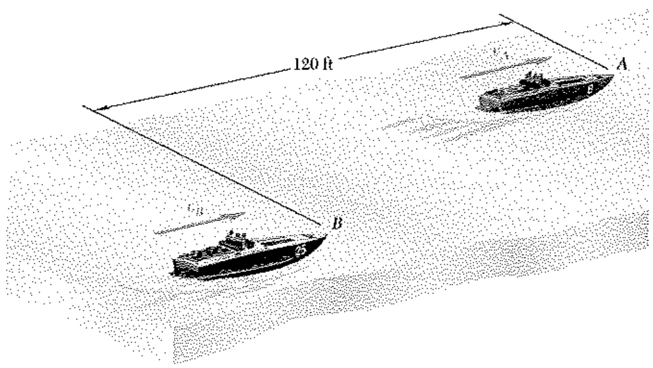
Eq. (1): $v_A = 35.2 + 1.8(15.05)$

$v_A = 62.29 \text{ ft/s}$ $\qquad v_A = 42.5 \text{ mi/h} \rightarrow$ ◀

Eq. (3): $v_B = 52.8 - 1.2(15.05)$

$v_B = 34.74 \text{ ft/s}$ $\qquad v_B = 23.7 \text{ mi/h} \rightarrow$ ◀

PROBLEM 11.42



In a boat race, boat A is leading boat B by 120 ft and both boats are traveling at a constant speed of 105 mi/h. At $t = 0$, the boats accelerate at constant rates. Knowing that when B passes A, $t = 8$ s and $v_A = 135$ mi/h, determine (*a*) the acceleration of A, (*b*) the acceleration of B.

SOLUTION

(*a*) We have

$$v_A = (v_A)_0 + a_A t$$

$$(v_A)_0 = 105 \text{ mi/h} = 154 \text{ ft/s}$$

At $t = 8$ s:

$$v_A = 135 \text{ mi/h} = 198 \text{ ft/s}$$

Then

$$198 \text{ ft/s} = 154 \text{ ft/s} + a_A(8 \text{ s})$$

or

$$a_A = 5.50 \text{ ft/s}^2 \blacktriangleleft$$

(*b*) We have

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2}a_A t^2 \qquad (x_A)_0 = 120 \text{ ft}$$

and

$$x_B = 0 + (v_B)_0 t + \frac{1}{2}a_B t^2 \qquad (v_B)_0 = 154 \text{ ft/s}$$

At $t = 8$ s:

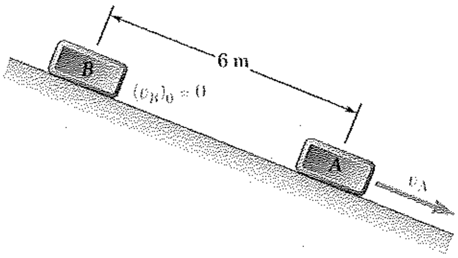
$$x_A = x_B$$

$$120 \text{ ft} + (154 \text{ ft/s})(8 \text{ s}) + \frac{1}{2}(5.50 \text{ ft/s}^2)(8 \text{ s})^2$$

$$= (154 \text{ ft/s})(8 \text{ s}) + \frac{1}{2}a_B(8 \text{ s})^2$$

or

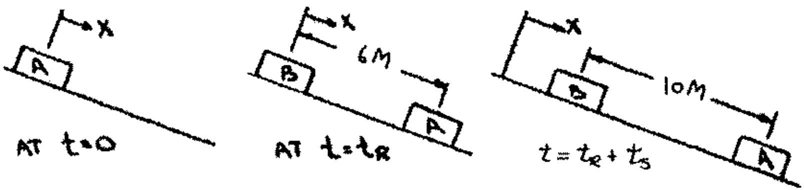
$$a_B = 9.25 \text{ ft/s}^2 \blacktriangleleft$$



PROBLEM 11.43

Boxes are placed on a chute at uniform intervals of time t_R and slide down the chute with uniform acceleration. Knowing that as any box B is released, the preceding box A has already slid 6 m and that 1 s later they are 10 m apart, determine (*a*) the value of t_R, (*b*) the acceleration of the boxes.

SOLUTION



Let $t_S = 1$ s be the time when the boxes are 30 ft apart.

Let $$a_A = a_B = a; \quad (x_A)_0 = (x_B)_0 = 0; \quad (v_A)_0 = (v_B)_0 = 0.$$

(*a*) For $$t > 0, \quad x_A = \frac{1}{2}at^2$$

For $$t > t_R, \quad x_B = \frac{1}{2}a(t - t_R)^2$$

At $$t = t_R, \quad x_A = 18 \text{ ft} \qquad 18 = \frac{1}{2}at_R^2 \tag{1}$$

At $$t = t_R + t_S, \qquad x_A - x_B = 30 \text{ ft}$$

$$30 = \frac{1}{2}a(t_R + t_S)^2 - \frac{1}{2}a(t_R + t_S - t_R)^2$$

$$= \frac{1}{2}at_R^2 + at_Rt_S + \frac{1}{2}at_S^2 - \frac{1}{2}at_S^2 = 18 + at_Rt_S$$

$$at_R = \frac{30 - 18}{t_S} = \frac{12}{1} = 12 \text{ ft/s} \tag{2}$$

Dividing Equation (1) by Eq. (2),

$$\frac{\frac{1}{2}at_R^2}{at_R} = \frac{1}{2}t_R = \frac{18}{12}$$

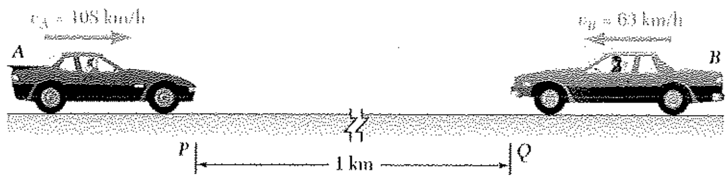
$t_R = 3.00$ s ◀

(*b*) Solving Eq. (2) for a,

$$a = \frac{12}{3} = 4 \text{ ft/s}^2$$

$a = 4.00$ ft/s^2 ◀

PROBLEM 11.44



Two automobiles A and B are approaching each other in adjacent highway lanes. At $t = 0$, A and B are 1 km apart, their speeds are $v_A = 108$ km/h and $v_B = 63$ km/h, and they are at Points P and Q, respectively. Knowing that A passes Point Q 40 s after B was there and that B passes Point P 42 s after A was there, determine (a) the uniform accelerations of A and B, (b) when the vehicles pass each other, (c) the speed of B at that time.

SOLUTION

(a) We have $$x_A = 0 + (v_A)_0 t + \frac{1}{2}a_A t^2 \quad (v_A)_0 = 108 \text{ km/h} = 30 \text{ m/s}$$

At $t = 40$ s: $$1000 \text{ m} = (30 \text{ m/s})(40 \text{ s}) + \frac{1}{2}a_A(40 \text{ s})^2$$

or $$a_A = -0.250 \text{ m/s}^2 \blacktriangleleft$$

Also, $$x_B = 0 + (v_B)_0 t + \frac{1}{2}a_B t^2 \quad (v_B)_0 = 63 \text{ km/h} = 17.5 \text{ m/s}$$

At $t = 42$ s: $$1000 \text{ m} = (17.5 \text{ m/s})(42 \text{ s}) + \frac{1}{2}a_B(42 \text{ s})^2$$

or $$a_B = 0.30045 \text{ m/s}^2 \qquad a_B = 0.300 \text{ m/s}^2 \blacktriangleleft$$

(b) When the cars pass each other

$$x_A + x_B = 1000 \text{ m}$$

Then $$(30 \text{ m/s})t_{AB} + \frac{1}{2}(-0.250 \text{ m/s})t_{AB}^2 + (17.5 \text{ m/s})t_{AB}$$
$$+\frac{1}{2}(0.30045 \text{ m/s}^2)t_{AB}^2 = 1000 \text{ m}$$

or $$0.05045t_{AB}^2 + 95t_{AB}^2 - 2000 = 0$$

Solving $$t = 20.822 \text{ s} \quad \text{and} \quad t = -1904 \text{ s}$$

$$t > 0 \Rightarrow t_{AB} = 20.8 \text{ s} \blacktriangleleft$$

PROBLEM 11.44 (Continued)

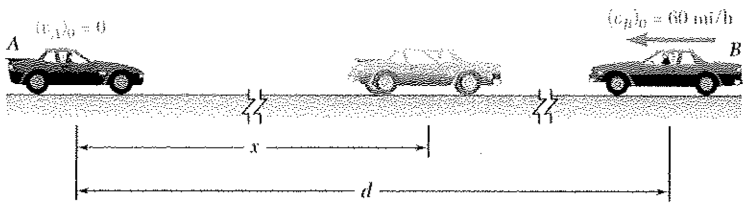
(*c*) We have $v_B = (v_B)_0 + a_B t$

At $t = t_{AB}$: $v_B = 17.5 \text{ m/s} + (0.30045 \text{ m/s}^2)(20.822 \text{ s})$

$= 23.756 \text{ m/s}$

or $v_B = 85.5 \text{ km/h}$ ◀

PROBLEM 11.45



Car A is parked along the northbound lane of a highway, and car B is traveling in the southbound lane at a constant speed of 60 mi/h. At $t = 0$, A starts and accelerates at a constant rate a_A, while at $t = 5$ s, B begins to slow down with a constant deceleration of magnitude $a_A/6$. Knowing that when the cars pass each other $x = 294$ ft and $v_A = v_B$, determine (*a*) the acceleration a_A, (*b*) when the vehicles pass each other, (*c*) the distance d between the vehicles at $t = 0$.

SOLUTION

For $t \geq 0$: $\quad v_A = 0 + a_A t$

$$x_A = 0 + 0 + \frac{1}{2} a_A t^2$$

$0 \leq t < 5$ s: $\quad x_B = 0 + (v_B)_0 t \quad (v_B)_0 = 60 \text{ mi/h} = 88 \text{ ft/s}$

At $t = 5$ s: $\quad x_B = (88 \text{ ft/s})(5 \text{ s}) = 440 \text{ ft}$

For $t \geq 5$ s: $\quad v_B = (v_B)_0 + a_B(t - s) \quad a_B = -\frac{1}{6} a_A$

$$x_B = (x_B)_S + (v_B)_0 (t - s) + \frac{1}{2} a_B (t - s)^2$$

Assume $t > 5$ s when the cars pass each other.

At that time (t_{AB}),

$v_A = v_B$: $\quad a_A t_{AB} = (88 \text{ ft/s}) - \frac{a_A}{6}(t_{AB} - 5)$

$x_A = 294$ ft: $\quad 294 \text{ ft} = \frac{1}{2} a_A t_{AB}^2$

PROBLEM 11.45 (Continued)

Then $$\frac{a_A\left(\frac{7}{6}t_{AB}-\frac{5}{6}\right)}{\frac{1}{2}a_A t_{AB}^2}=\frac{88}{294}$$

or $$44t_{AB}^2-343t_{AB}+245=0$$

Solving $$t_{AB}=0.795\text{ s}\quad\text{and}\quad t_{AB}=7.00\text{ s}$$

(*a*) With $t_{AB}>5$ s, $$294\text{ ft}=\frac{1}{2}a_A(7.00\text{ s})^2$$

or $a_A=12.00\text{ ft/s}^2$ ◀

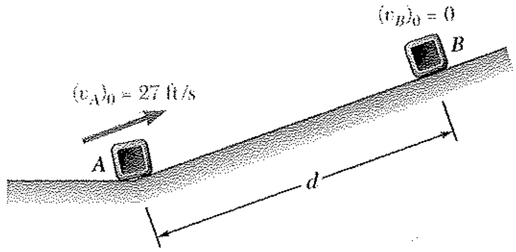
(*b*) From above $t_{AB}=7.00\text{ s}$ ◀

Note: An acceptable solution cannot be found if it is assumed that $t_{AB}\le 5$ s.

(*c*) We have
$$\begin{aligned}d&=x+(x_B)_{t_{AB}}\\&=294\text{ ft}+[440\text{ ft}+(88\text{ ft/s})(7.00\text{ s})\text{ s}\\&\quad+\frac{1}{2}\left(-\frac{1}{6}\times 12.00\text{ ft/s}^2\right)(7.00\text{ s})^2\text{s}^2\end{aligned}$$

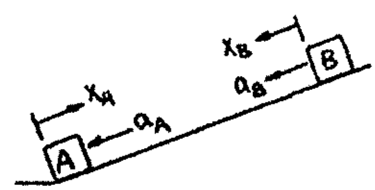
or $d=906\text{ ft}$ ◀

PROBLEM 11.46



Two blocks A and B are placed on an incline as shown. At $t = 0$, A is projected up the incline with an initial velocity of 27 ft/s and B is released from rest. The blocks pass each other 1 s later, and B reaches the bottom of the incline when $t = 3.4$ s. Knowing that the maximum distance from the bottom of the incline reached by block A is 21 ft and that the accelerations of A and B (due to gravity and friction) are constant and are directed down the incline, determine (a) the accelerations of A and B, (b) the distance d, (c) the speed of A when the blocks pass each other.

SOLUTION



(a) We have $\quad v_A^2 = (v_A)_0^2 + 2a_A[x_A - 0]$

When $\quad x_A = (x_A)_{\max}, \quad v_A = 0$

Then $\quad 0 = (27 \text{ ft/s})^2 + 2a_A(21 \text{ ft})$

or $\quad a_A = -17.3571 \text{ ft/s}^2$

or $\quad \mathbf{a}_A = 17.36 \text{ ft/s}^2 \swarrow$ ◀

Now $\quad x_A = 0 + (v_A)_0 t + \frac{1}{2}a_A t^2$

and $\quad x_B = 0 + 0t + \frac{1}{2}a_B t^2$

At $t = 1$ s, the blocks pass each other.

$$(x_A)_1 + (x_B)_1 = d$$

At $t = 3.4$ s, $x_B = d$:

Thus $\quad (x_A)_1 + (x_B)_1 = (x_B)_{3.4}$

or

$$\left[(27 \text{ ft/s})(1 \text{ s}) + \frac{1}{2}(-17.3571 \text{ ft/s}^2)(1 \text{ s})^2\right] + \left[\frac{1}{2}a_B(1 \text{ s})^2\right] = \frac{1}{2}a_B(3.4 \text{ s})^2$$

or $\quad a_B = 3.4700 \text{ ft/s}^2$ $\qquad \mathbf{a}_B = 3.47 \text{ ft/s}^2 \swarrow$ ◀

PROBLEM 11.46 (Continued)

(*b*) At $t = 3.4$ s, $x_B = d$: $$d = \frac{1}{2}(3.4700 \text{ ft/s}^2)(3.4 \text{ s})^2$$

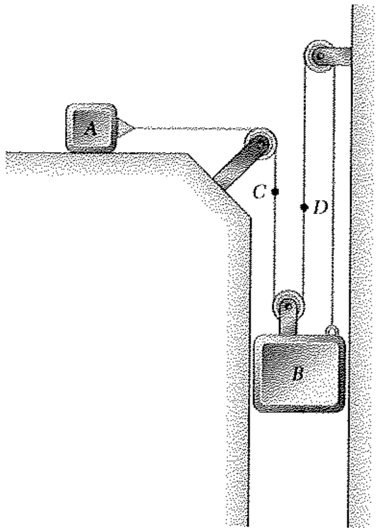
or $$d = 20.1 \text{ ft} \blacktriangleleft$$

(*c*) We have $$v_A = (v_A)_0 + a_A t$$

At $t = 1$ s: $$v_A = 27 \text{ ft/s} + (-17.3571 \text{ ft/s})(1 \text{ s})$$

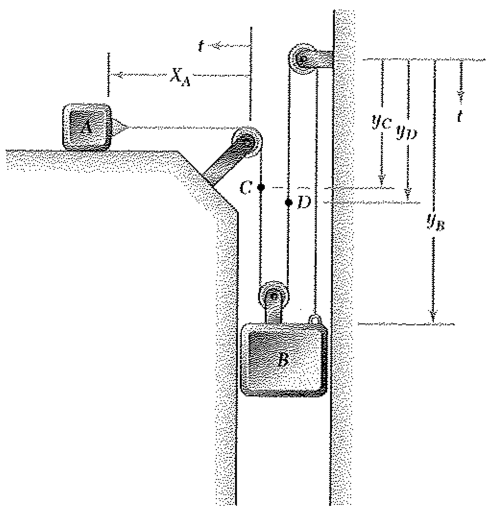
or $$v_A = 9.64 \text{ ft/s} \blacktriangleleft$$

PROBLEM 11.47



Slider block A moves to the left with a constant velocity of 6 m/s. Determine (*a*) the velocity of block B, (*b*) the velocity of portion D of the cable, (*c*) the relative velocity of portion C of the cable with respect to portion D.

SOLUTION



From the diagram, we have

$$x_A + 3y_B = \text{constant}$$

Then $\quad v_A + 3v_B = 0 \qquad (1)$

and $\quad a_A + 3a_B = 0 \qquad (2)$

(*a*) Substituting into Eq. (1) $\quad 6 \text{ m/s} + 3v_B = 0$

or $\quad \mathbf{v}_B = 2 \text{ m/s} \uparrow$ ◀

(*b*) From the diagram $\quad y_B + y_D = \text{constant}$

Then $\quad v_B + v_D = 0$

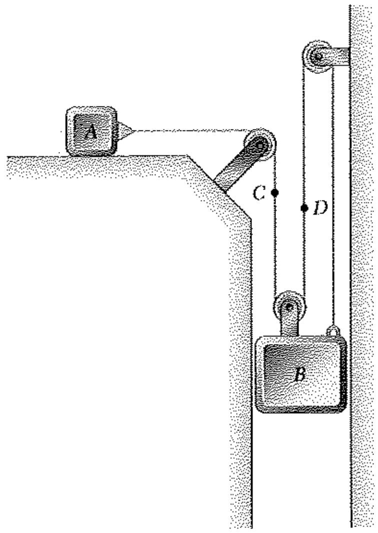
$$\mathbf{v}_D = 2 \text{ m/s} \downarrow \text{ ◀}$$

(*c*) From the diagram $\quad x_A + y_C = \text{constant}$

Then $\quad v_A + v_C = 0 \qquad v_C = -6 \text{ m/s}$

Now $\quad v_{C/D} = v_C - v_D = (-6 \text{ m/s}) - (2 \text{ m/s}) = -8 \text{ m/s}$

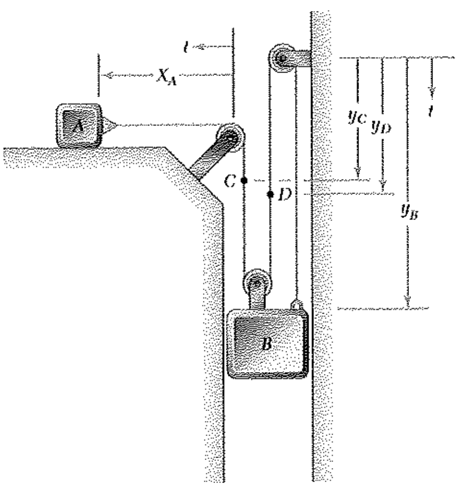
$$\mathbf{v}_{C/D} = 8 \text{ m/s} \uparrow \text{ ◀}$$



PROBLEM 11.48

Block B starts from rest and moves downward with a constant acceleration. Knowing that after slider block A has moved 400 mm its velocity is 4 m/s, determine (*a*) the accelerations of A and B, (*b*) the velocity and the change in position of B after 2 s.

SOLUTION



From the diagram, we have

$$x_A + 3y_B = \text{constant}$$

Then $\quad v_A + 3v_B = 0 \qquad (1)$

and $\quad a_A + 3a_B = 0 \qquad (2)$

(*a*) Eq. (2): $a_A + 3a_B = 0$ and $\mathbf{a}_B$ is constant and positive $\Rightarrow \mathbf{a}_A$ is constant and negative

Also, Eq. (1) and $(v_B)_0 = 0 \Rightarrow (v_A)_0 = 0$

Then $\quad v_A^2 = 0 + 2a_A[x_A - (x_A)_0]$

When $|\Delta x_A| = 0.4$ m: $\quad (4 \text{ m/s})^2 = 2a_A(0.4 \text{ m})$

or $\quad \mathbf{a}_A = 20 \text{ m/s}^2 \rightarrow$ ◀

Then, substituting into Eq. (2):

$$-20 \text{ m/s}^2 + 3a_B = 0$$

or $\quad a_B = \dfrac{20}{3} \text{ m/s}^2 \qquad \mathbf{a}_B = 6.67 \text{ m/s}^2 \downarrow$ ◀

(*b*) We have $v_B = 0 + a_B t$

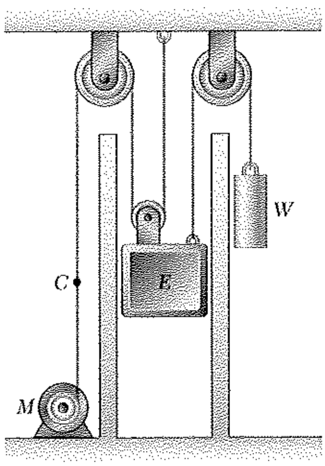
At $t = 2$ s: $v_B = \left(\frac{20}{3}\ \text{m/s}^2\right)(2\ \text{s})$

or $\mathbf{v}_B = 13.33\ \text{m/s} \downarrow$ ◀

Also $y_B = (y_B)_0 + 0 + \frac{1}{2} a_B t^2$

At $t = 2$ s: $y_B - (y_B)_0 = \frac{1}{2}\left(\frac{20}{3}\ \text{m/s}^2\right)(2\ \text{s})^2$

or $\mathbf{y}_B - (\mathbf{y}_B)_0 = 13.33\ \text{m} \downarrow$ ◀

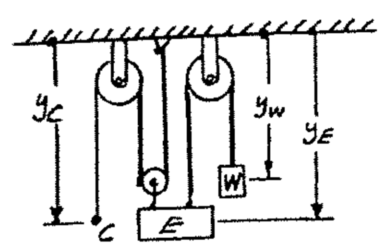


PROBLEM 11.49

The elevator shown in the figure moves downward with a constant velocity of 15 ft/s. Determine (*a*) the velocity of the cable *C*, (*b*) the velocity of the counterweight *W*, (*c*) the relative velocity of the cable *C* with respect to the elevator, (*d*) the relative velocity of the counterweight *W* with respect to the elevator.

SOLUTION

Choose the positive direction downward.



(*a*) *Velocity of cable C.*

$$y_C + 2y_E = \text{constant}$$

$$v_C + 2v_E = 0$$

But, $$v_E = 15 \text{ ft/s}$$

or $$v_C = -2v_E = -30 \text{ ft/s}$$

$\mathbf{v}_C = 30.0 \text{ ft/s} \uparrow$ ◀

(*b*) *Velocity of counterweight W.*

$$y_W + y_E = \text{constant}$$

$$v_W + v_E = 0 \quad v_W = -v_E = -15 \text{ ft/s}$$

$\mathbf{v}_W = 15.00 \text{ ft/s} \uparrow$ ◀

(*c*) *Relative velocity of C with respect to E.*

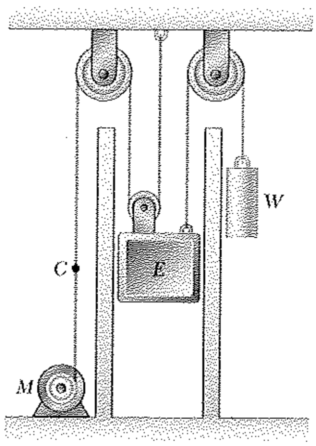
$$v_{C/E} = v_C - v_E = (-30 \text{ ft/s}) - (+15 \text{ ft/s}) = -45 \text{ ft/s}$$

$\mathbf{v}_{C/E} = 45.0 \text{ ft/s} \uparrow$ ◀

(*d*) *Relative velocity of W with respect to E.*

$$v_{W/E} = v_W - v_E = (-15 \text{ ft/s}) - (15 \text{ ft/s}) = -30 \text{ ft/s}$$

$\mathbf{v}_{W/E} = 30.0 \text{ ft/s} \uparrow$ ◀

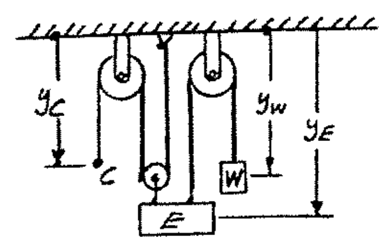


PROBLEM 11.50

The elevator shown starts from rest and moves upward with a constant acceleration. If the counterweight W moves through 30 ft in 5 s, determine (a) the accelerations of the elevator and the cable C, (b) the velocity of the elevator after 5 s.

SOLUTION

We choose Positive direction downward for motion of counterweight.



$$y_W = \frac{1}{2}a_W t^2$$

At $t = 5$ s,

$$y_W = 30 \text{ ft}$$

$$30 \text{ ft} = \frac{1}{2}a_W(5 \text{ s})^2$$

$$a_W = 2.4 \text{ ft/s}^2 \qquad \mathbf{a}_W = 2.4 \text{ ft/s}^2 \downarrow$$

(a) *Accelerations of E and C.*

Since $\quad y_W + y_E = \text{constant} \quad v_W + v_E = 0, \quad \text{and} \quad a_W + a_E = 0$

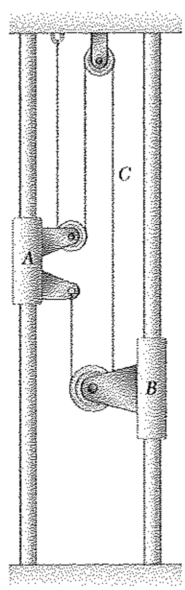
Thus: $\quad a_E = -a_W = -(2.4 \text{ ft/s}^2), \qquad \mathbf{a}_E = 2.40 \text{ ft/s}^2 \uparrow$ ◀

Also, $\quad y_C + 2y_E = \text{ constant}, \quad v_C + 2v_E = 0, \quad \text{and} \quad a_C + 2a_E = 0$

Thus: $\quad a_C = -2a_E = -2(-2.4 \text{ ft/s}^2) = +4.8 \text{ ft/s}^2, \qquad \mathbf{a}_C = 4.80 \text{ ft/s}^2 \downarrow$ ◀

(b) *Velocity of elevator after 5 s.*

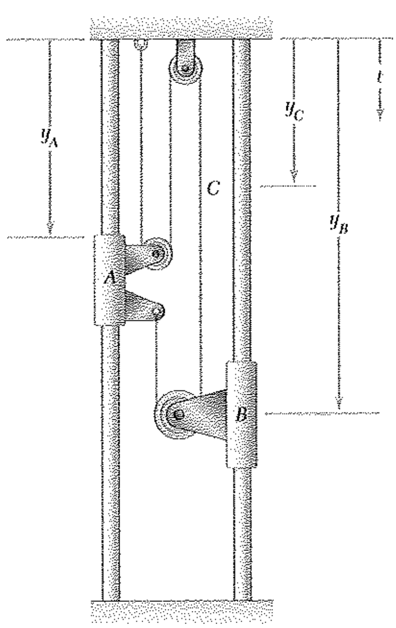
$$v_E = (v_E)_0 + a_E t = 0 + (-2.4 \text{ ft/s}^2)(5 \text{ s}) = -12 \text{ ft/s} \qquad (\mathbf{v}_E)_5 = 12.00 \text{ ft/s} \uparrow \text{ ◀}$$



PROBLEM 11.51

Collar A starts from rest and moves upward with a constant acceleration. Knowing that after 8 s the relative velocity of collar B with respect to collar A is 24 in./s, determine (*a*) the accelerations of A and B, (*b*) the velocity and the change in position of B after 6 s.

SOLUTION



From the diagram

$$2y_A + y_B + (y_B - y_A) = \text{constant}$$

Then $$v_A + 2v_B = 0 \qquad (1)$$

and $$a_A + 2a_B = 0 \qquad (2)$$

(*a*) Eq. (1) and $(v_A)_0 = 0 \Rightarrow (v_B)_0$

Also, Eq. (2) and $\mathbf{a}_A$ is constant and negative $\Rightarrow \mathbf{a}_B$ is constant and positive

Then $$v_A = 0 + a_A t \qquad v_B = 0 + a_B t$$

Now $$v_{B/A} = v_B - v_A = (a_B - a_A)t$$

From Eq. (2) $$a_B = -\frac{1}{2}a_A$$

So that $$v_{B/A} = -\frac{3}{2}a_A t$$

PROBLEM 11.51 (Continued)

At $t = 8$ s: $$24 \text{ in./s} = -\frac{3}{2}a_A(8 \text{ s})$$

or $$\mathbf{a}_A = 2 \text{ in./s}^2 \uparrow \blacktriangleleft$$

and then $$a_B = -\frac{1}{2}(-2 \text{ in./s}^2)$$

or $$\mathbf{a}_B = 1 \text{ in./s}^2 \uparrow \blacktriangleleft$$

(*b*) At $t = 6$ s: $$v_B = (1 \text{ in./s}^2)(6 \text{ s})$$

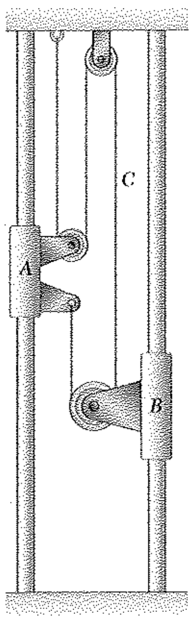
or $$\mathbf{v}_B = 6 \text{ in./s} \downarrow \blacktriangleleft$$

Now $$y_B = (y_B)_0 + 0 + \frac{1}{2}a_B t^2$$

At $t = 6$ s: $$y_B - (y_B)_0 = \frac{1}{2}(1 \text{ in./s}^2)(6 \text{ s})^2$$

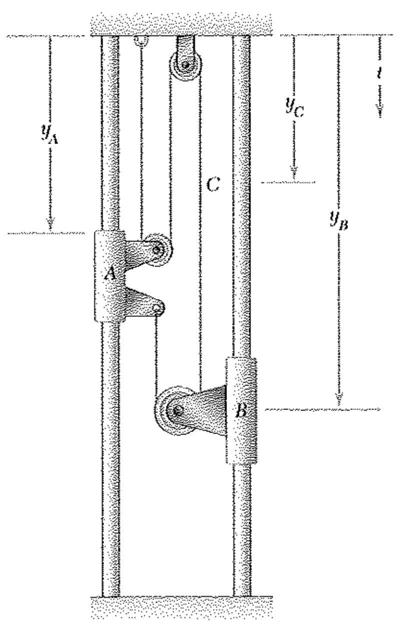
or $$\mathbf{y}_B - (\mathbf{y}_B)_0 = 18 \text{ in.} \downarrow \blacktriangleleft$$

PROBLEM 11.52



In the position shown, collar B moves downward with a velocity of 12 in./s. Determine (*a*) the velocity of collar A, (*b*) the velocity of portion C of the cable, (*c*) the relative velocity of portion C of the cable with respect to collar B.

SOLUTION



From the diagram

$$2y_A + y_B = (y_B - y_A) = \text{constant}$$

Then $\quad v_A + 2v_B = 0 \quad (1)$

and $\quad a_A + 2a_B = 0 \quad (2)$

(*a*) Substituting into Eq. (1) $\quad v_A + 2(12 \text{ in./s}) = 0$

or $\quad \mathbf{v}_A = 24 \text{ in./s} \uparrow$ ◄

(*b*) From the diagram $\quad 2y_A + y_C = \text{constant}$

Then $\quad 2v_A + v_C = 0$

Substituting $\quad 2(-24 \text{ in./s}) + v_C = 0$

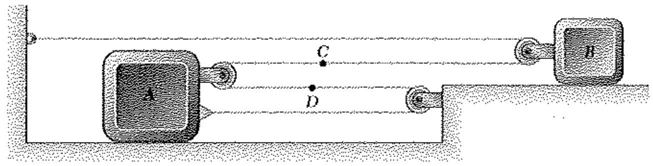
or $\quad \mathbf{v}_C = 48 \text{ in./s} \downarrow$ ◄

(*c*) We have $\quad v_{C/B} = v_C - v_B$

$$= (48 \text{ in./s}) - (12 \text{ in./s})$$

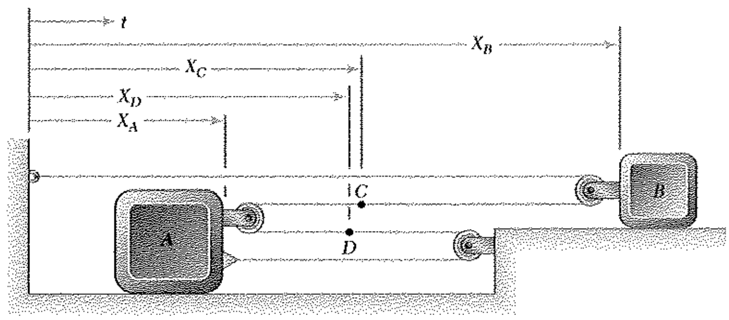
or $\quad \mathbf{v}_{C/B} = 36 \text{ in./s} \downarrow$ ◄

PROBLEM 11.53



Slider block B moves to the right with a constant velocity of 300 mm/s. Determine (a) the velocity of slider block A, (b) the velocity of portion C of the cable, (c) the velocity of portion D of the cable, (d) the relative velocity of portion C of the cable with respect to slider block A.

SOLUTION



From the diagram $\quad x_B + (x_B - x_A) - 2x_A = \text{constant}$

Then $\quad 2v_B - 3v_A = 0 \qquad (1)$

and $\quad 2a_B - 3a_A = 0 \qquad (2)$

Also, we have $\quad -x_B - x_A = \text{constant}$

Then $\quad v_D + v_A = 0 \qquad (3)$

(a) Substituting into Eq. (1) $\quad 2(300 \text{ mm/s}) - 3v_A = 0$

or $\quad \mathbf{v}_A = 200 \text{ mm/s} \rightarrow$ ◀

(b) From the diagram $\quad x_B + (x_B - x_C) = \text{constant}$

Then $\quad 2v_B - v_C = 0$

Substituting $\quad 2(300 \text{ mm/s}) - v_C = 0$

or $\quad \mathbf{v}_C = 600 \text{ mm/s} \rightarrow$ ◀

PROBLEM 11.53 (Continued)

(*c*) From the diagram $(x_C - x_A) + (x_B - x_A) = \text{constant}$

Then $v_C - 2v_A + v_D = 0$

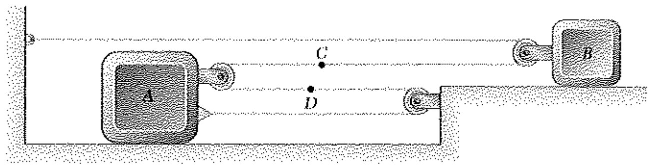
Substituting $600 \text{ mm/s} - 2(200 \text{ mm/s}) + v_D = 0$

or $\mathbf{v}_D = 200 \text{ mm/s} \leftarrow$ ◀

(*d*) We have

$$v_{C/A} = v_C - v_A$$
$$= 600 \text{ mm/s} - 200 \text{ mm/s}$$

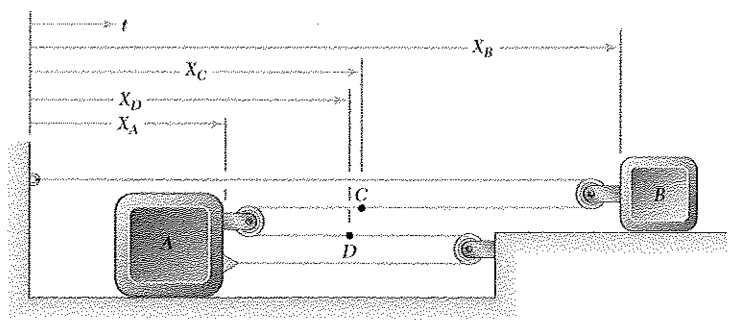
or $\mathbf{v}_{C/A} = 400 \text{ mm/s} \rightarrow$ ◀



PROBLEM 11.54

At the instant shown, slider block B is moving with a constant acceleration, and its speed is 150 mm/s. Knowing that after slider block A has moved 240 mm to the right its velocity is 60 mm/s, determine (a) the accelerations of A and B, (b) the acceleration of portion D of the cable, (c) the velocity and change in position of slider block B after 4 s.

SOLUTION



From the diagram $\qquad x_B + (x_B - x_A) - 2x_A = \text{constant}$

Then $\qquad 2v_B - 3v_A = 0 \qquad (1)$

and $\qquad 2a_B - 3a_A = 0 \qquad (2)$

(a) First observe that if block A moves to the right, $\mathbf{v}_A \rightarrow$ and Eq. (1) $\Rightarrow \mathbf{v}_B \rightarrow$. Then, using Eq. (1) at $t = 0$

$$2(150 \text{ mm/s}) - 3(v_A)_0 = 0$$

or $\qquad (v_A)_0 = 100 \text{ mm/s}$

Also, Eq. (2) and $a_B =$ constant $\Rightarrow a_A =$ constant

Then $\qquad v_A^2 = (v_A)_0^2 + 2a_A[x_A - (x_A)_0]$

When $x_A - (x_A)_0 = 240$ mm:

$$(60 \text{ mm/s})^2 = (100 \text{ mm/s})^2 + 2a_A(240 \text{ mm})$$

or $\qquad a_A = -\dfrac{40}{3} \text{ mm/s}^2$

or $\qquad \mathbf{a}_A = 13.33 \text{ mm/s}^2 \leftarrow$ ◀

PROBLEM 11.54 (Continued)

Then, substituting into Eq. (2)

$$2a_B - 3\left(-\frac{40}{3}\ \text{mm/s}^2\right) = 0$$

or $\qquad a_B = -20\ \text{mm/s}^2 \qquad \mathbf{a}_B = 20.0\ \text{mm/s}^2 \leftarrow$ ◀

(*b*) From the solution to Problem 11.53

$$v_D + v_A = 0$$

Then $\qquad a_D + a_A = 0$

Substituting $\qquad a_D + \left(-\frac{40}{3}\ \text{mm/s}^2\right) = 0$

or $\qquad \mathbf{a}_D = 13.33\ \text{mm/s}^2 \rightarrow$ ◀

(*c*) We have $\qquad v_B = (v_B)_0 + a_B t$

At $t = 4$ s: $\qquad v_B = 150\ \text{mm/s} + (-20.0\ \text{mm/s}^2)(4\ \text{s})$

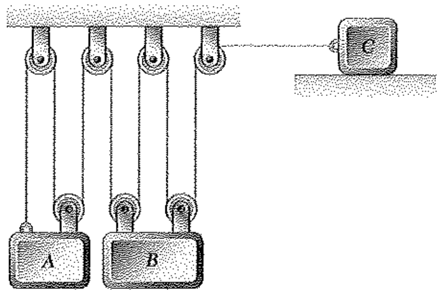
or $\qquad \mathbf{v}_B = 70.0\ \text{mm/s} \rightarrow$ ◀

Also $\qquad y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$

At $t = 4$ s:

$$y_B - (y_B)_0 = (150\ \text{mm/s})(4\ \text{s}) + \frac{1}{2}(-20.0\ \text{mm/s}^2)(4\ \text{s})^2$$

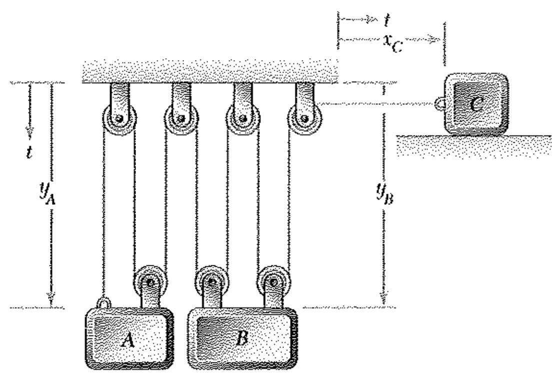
or $\qquad \mathbf{y}_B - (\mathbf{y}_B)_0 = 440\ \text{mm} \rightarrow$ ◀



PROBLEM 11.55

Block B moves downward with a constant velocity of 20 mm/s. At $t = 0$, block A is moving upward with a constant acceleration, and its velocity is 30 mm/s. Knowing that at $t = 3$ s slider block C has moved 57 mm to the right, determine (a) the velocity of slider block C at $t = 0$, (b) the accelerations of A and C, (c) the change in position of block A after 5 s.

SOLUTION



From the diagram

$$3y_A + 4y_B + x_C = \text{constant}$$

Then $\quad 3v_A + 4v_B + v_C = 0 \qquad (1)$

and $\quad 3a_A + 4a_B + a_C = 0 \qquad (2)$

<u>Given</u>: $\quad \mathbf{v}_B = 20 \text{ mm/s} \downarrow;$

$$(\mathbf{v}_A)_0 = 30 \text{ mm/s} \uparrow$$

(a) Substituting into Eq. (1) at $t = 0$

$$3(-30 \text{ mm/s}) + 4(20 \text{ mm/s}) + (v_C)_0 = 0$$

$$v_C = 10 \text{ mm/s} \qquad \text{or} \qquad (\mathbf{v}_C)_0 = 10 \text{ mm/s} \rightarrow \blacktriangleleft$$

(b) We have $\quad x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$

At $t = 3$ s: $\quad 57 \text{ mm} = (10 \text{ mm/s})(3 \text{ s}) + \frac{1}{2} a_C (3 \text{ s})^2$

$$a_C = 6 \text{ mm/s}^2 \qquad \text{or} \qquad \mathbf{a}_C = 6 \text{ mm/s}^2 \rightarrow \blacktriangleleft$$

Now $\quad \mathbf{v}_B = \text{constant} \rightarrow a_B = 0$

PROBLEM 11.55 (Continued)

Then, substituting into Eq. (2)

$$3a_A + 4(0) + (6 \text{ mm/s}^2) = 0$$

$$a_A = -2 \text{ mm/s}^2 \qquad \text{or} \qquad \mathbf{a}_A = 2 \text{ mm/s}^2 \uparrow \blacktriangleleft$$

(*c*) We have

$$y_A = (y_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

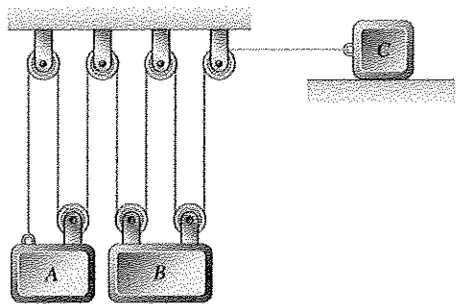
At $t = 5$ s:

$$y_A - (y_A)_0 = (-30 \text{ mm/s})(5 \text{ s}) + \frac{1}{2}(-2 \text{ mm/s}^2)(5 \text{ s})^2$$

$$= -175 \text{ mm}$$

or

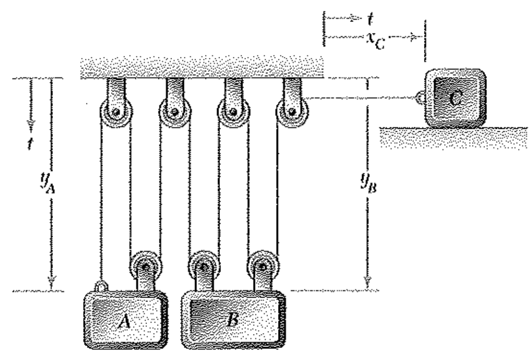
$$\mathbf{y}_A - (\mathbf{y}_A)_0 = 175 \text{ mm} \uparrow \blacktriangleleft$$



PROBLEM 11.56

Block B starts from rest, block A moves with a constant acceleration, and slider block C moves to the right with a constant acceleration of 75 mm/s^2. Knowing that at $t = 2$ s the velocities of B and C are 480 mm/s downward and 280 mm/s to the right, respectively, determine (a) the accelerations of A and B, (b) the initial velocities of A and C, (c) the change in position of slider C after 3 s.

SOLUTION



From the diagram

$$3y_A + 4y_B + x_C = \text{constant}$$

Then $$3v_A + 4v_B + v_C = 0 \qquad (1)$$

and $$3a_A + 4a_B + a_C = 0 \qquad (2)$$

Given: $$(v_B) = 0,$$

$$a_A = \text{constent}$$

$$(\mathbf{a}_C) = 75 \text{ mm/s}^2 \longrightarrow$$

At $t = 2$ s, $$\mathbf{v}_B = 480 \text{ mm/s} \downarrow$$
$$\mathbf{v}_C = 280 \text{ mm/s} \longrightarrow$$

(a) Eq. (2) and $a_A = \text{constant}$ and $a_C = \text{constant} \Rightarrow a_B = \text{constant}$

Then $$v_B = 0 + a_B t$$

At $t = 2$ s: $$480 \text{ mm/s} = a_B(2 \text{ s})$$

$$a_B = 240 \text{ mm/s}^2 \qquad \text{or} \qquad \mathbf{a}_B = 240 \text{ mm/s}^2 \downarrow \blacktriangleleft$$

Substituting into Eq. (2)

$$3a_A + 4(240 \text{ mm/s}^2) + (75 \text{ mm/s}^2) = 0$$

$$a_A = -345 \text{ mm/s} \qquad \text{or} \qquad \mathbf{a}_A = 345 \text{ mm/s}^2 \uparrow \blacktriangleleft$$

PROBLEM 11.56 (Continued)

(*b*) We have $$v_C = (v_C)_0 + a_C t$$

At $t = 2$ s: $$280 \text{ mm/s} = (v_C)_0 + (75 \text{ mm/s})(2 \text{ s})$$

$$v_C = -130 \text{ mm/s} \qquad \text{or} \qquad (\mathbf{v}_C)_0 = 130 \text{ mm/s} \rightarrow \blacktriangleleft$$

Then, substituting into Eq. (1) at $t = 0$

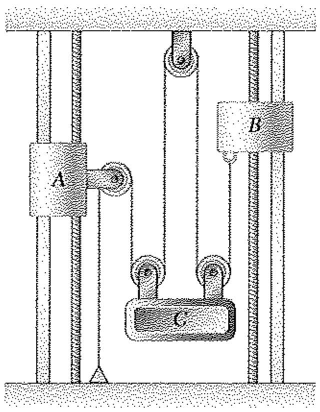
$$3(v_A)_0 + 4(0) + (130 \text{ mm/s}) = 0$$

$$v_A = -43.3 \text{ mm/s} \qquad \text{or} \qquad (\mathbf{v}_A)_0 = 43.3 \text{ mm/s} \uparrow \blacktriangleleft$$

(*c*) We have $$x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$$

At $t = 3$ s: $$x_C - (x_C)_0 = (130 \text{ mm/s})(3 \text{ s}) + \frac{1}{2}(75 \text{ mm/s}^2)(3 \text{ s})^2$$

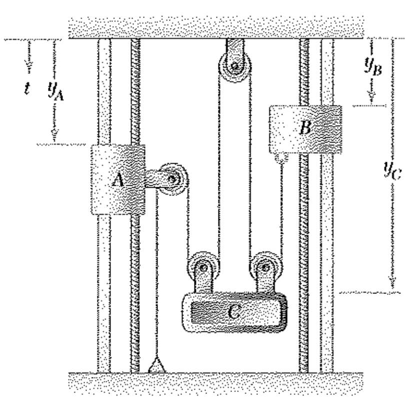
$$= -728 \text{ mm} \qquad \text{or} \qquad \mathbf{x}_C - (\mathbf{x}_C)_0 = 728 \text{ mm} \rightarrow \blacktriangleleft$$



PROBLEM 11.57

Collar A starts from rest at $t = 0$ and moves downward with a constant acceleration of 7 in./s^2. Collar B moves upward with a constant acceleration, and its initial velocity is 8 in./s. Knowing that collar B moves through 20 in. between $t = 0$ and $t = 2$ s, determine (a) the accelerations of collar B and block C, (b) the time at which the velocity of block C is zero, (c) the distance through which block C will have moved at that time.

SOLUTION



From the diagram

$$-y_A + (y_C - y_A) + 2y_C + (y_C - y_B) = \text{constant}$$

Then $\qquad -2v_A - v_B + 4v_C = 0 \qquad (1)$

and $\qquad -2a_A - a_B + 4a_C = 0 \qquad (2)$

Given: $\qquad (v_A)_0 = 0$

$\qquad (\mathbf{a}_A) = 7 \text{ in./s}^2 \downarrow$

$\qquad (\mathbf{v}_B)_0 = 8 \text{ in./s} \uparrow$

$\qquad \mathbf{a}_B = \text{constant} \uparrow$

At $t = 2$ s $\qquad y - (y_B)_0 = 20 \text{ in.} \uparrow$

(a) We have $\qquad y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$

At $t = 2$ s: $\qquad -20 \text{ in.} = (-8 \text{ in./s})(2 \text{ s}) + \frac{1}{2} a_B (2 \text{ s})^2$

$a_B = -4 \text{ in./s}^2$ or $\qquad \mathbf{a}_B = 2 \text{ in./s}^2 \uparrow$ ◀

Then, substituting into Eq. (2)

$$-2(7 \text{ in./s}^2) - (-2 \text{ in./s}^2) + 4a_C = 0$$

$a_C = 3 \text{ in./s}^2$ or $\qquad \mathbf{a}_C = 3 \text{ in./s}^2 \downarrow$ ◀

PROBLEM 11.57 (Continued)

(*b*) Substituting into Eq. (1) at $t=0$

$$-2(0)-(-8 \text{ in./s})+4(v_C)_0=0 \quad \text{or} \quad (v_C)_0=-2 \text{ in./s}$$

Now $$v_C=(v_C)_0+a_C t$$

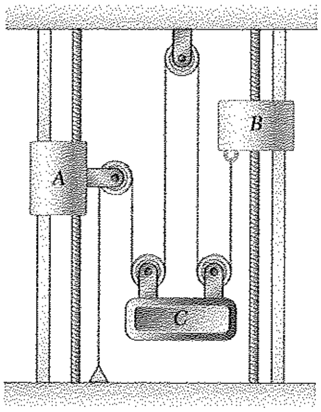
When $v_C=0$: $$0=(-2 \text{ in./s})+(3 \text{ in./s}^2)t$$

or $$t=\frac{2}{3} \text{ s}$$ $t=0.667 \text{ s}$ ◀

(*c*) We have $$y_C=(y_C)_0+(v_C)_0 t+\frac{1}{2}a_C t^2$$

At $t=\frac{2}{3}$ s: $$y_C-(y_C)_0=(-2 \text{ in./s})\left(\frac{2}{3} \text{ s}\right)+\frac{1}{2}(3 \text{ in./s}^2)\left(\frac{2}{3} \text{ s}\right)^2$$

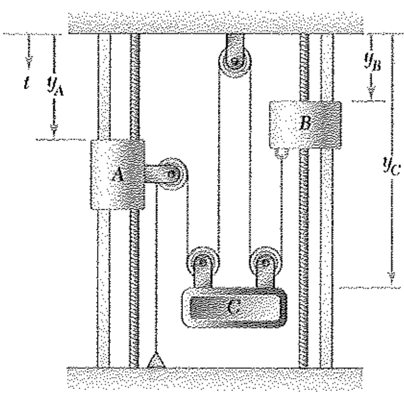
$$=-0.667 \text{ in.} \qquad \text{or} \qquad \mathbf{y}_C-(\mathbf{y}_C)_0=0.667 \text{ in.}\uparrow$$ ◀



PROBLEM 11.58

Collars A and B start from rest, and collar A moves upward with an acceleration of $3t^2$ in./s^2. Knowing that collar B moves downward with a constant acceleration and that its velocity is 8 in./s after moving 32 in., determine (*a*) the acceleration of block C, (*b*) the distance through which block C will have moved after 3 s.

SOLUTION



From the diagram

$$-y_A + (y_C - y_A) + 2y_C + (y_C - y_B) = \text{constant}$$

Then $\qquad -2v_A - v_B + 4v_C = 0 \qquad (1)$

and $\qquad -2a_A - a_B + 4a_C = 0 \qquad (2)$

Given: $\qquad (v_A)_0 = 0$

$(v_B)_0 = 0$

$\mathbf{a}_A = 3t^2 \text{ in./s}^2 \uparrow$

$\mathbf{a}_B = \text{constant} \downarrow$

When $\qquad \mathbf{y}_B - (\mathbf{y}_B)_0 = 32 \text{ in.} \downarrow,$

$\mathbf{v}_B = 8 \text{ in./s}$

(*a*) We have $\qquad v_B^2 = 0 + 2a_B[y_B - (y_B)_0]$

When $y_B - (y_B)_0 = 32$ in.: $\qquad (8 \text{ in./s})^2 = 2a_B(32 \text{ in.})$

or $\qquad a_B = 1 \text{ in./s}^2$

Then, substituting into Eq. (2)

$$-2(-3t^2 \text{ in./s}^2) - (1 \text{ in./s}^2) + 4a_C = 0$$

or $\qquad a_C = \frac{1}{4}(1 - 6t^2) \text{ in./s}^2$ ◀

(*b*) Substituting into Eq. (1) at $t = 0$

$$-2(0) - (0) + 4(v_C)_0 = 0 \quad \text{or} \quad (v_C)_0 = 0$$

Now $\qquad \frac{dv_C}{dt} = a_C = \frac{1}{4}(1 - 6t^2)$

PROBLEM 11.58 (Continued)

At $t=0,\ v_C=0$: $\displaystyle\int_0^{v_C} dv_C = \int_0^t \frac{1}{4}(1-6t^2)dt$

or $v_C = \dfrac{1}{4}(t-2t^3)$

Thus, $v_C = 0$

At $\dfrac{1}{4}t(1-2t^2)=0$

or $t=0, \quad t=\dfrac{1}{\sqrt{2}}$ s

Therefore, block C initially moves downward $(v_C > 0)$ and then moves upward $(v_C < 0)$.

Now $\dfrac{dy_C}{dt} = v_C = \dfrac{1}{4}(t-2t^3)$

At $t=0,\ y_C=(y_C)_0$: $\displaystyle\int_{(y_C)_0}^{y_C} dy_C = \int_0^t \frac{1}{4}(t-2t^3)dt$

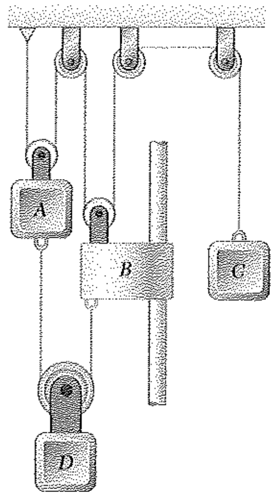
or $y_C-(y_C)_0 = \dfrac{1}{8}(t^2-t^4)$

At $t=\frac{1}{\sqrt{2}}$ s: $y_C-(y_C)_0 = \dfrac{1}{8}\left[\left(\dfrac{1}{\sqrt{2}}\right)^2 - \left(\dfrac{1}{\sqrt{2}}\right)^4\right] = \dfrac{1}{32}$ in.

At $t=3$ s: $y_C-(y_C)_0 = \dfrac{1}{8}[(3)^2-(3)^4] = -9$ in.

Total distance traveled $=\left(\dfrac{1}{32}\right)+\left|-9-\dfrac{1}{32}\right| = 9\dfrac{1}{16}$ in.

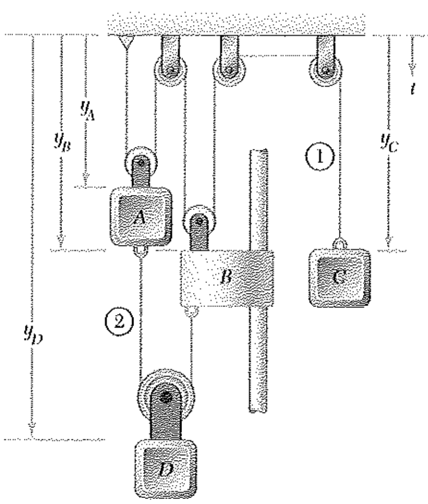
$= 9.06$ in. ◀



PROBLEM 11.59

The system shown starts from rest, and each component moves with a constant acceleration. If the relative acceleration of block C with respect to collar B is $60\ \text{mm/s}^2$ upward and the relative acceleration of block D with respect to block A is $110\ \text{mm/s}^2$ downward, determine (a) the velocity of block C after 3 s, (b) the change in position of block D after 5 s.

SOLUTION



From the diagram

Cable 1: $\qquad 2y_A + 2y_B + y_C = \text{constant}$

Then $\qquad 2v_A + 2v_B + v_C = 0 \qquad (1)$

and $\qquad 2a_A + 2a_B + a_C = 0 \qquad (2)$

Cable 2: $\qquad (y_D - y_A) + (y_D - y_B) = \text{constant}$

Then $\qquad -v_A - v_B + 2v_D = 0 \qquad (3)$

and $\qquad -a_A - a_B + 2a_D = 0 \qquad (4)$

Given: At $t = 0$, $v = 0$; all accelerations constant;
$a_{C/B} = 60\ \text{mm/s}^2\uparrow,\ a_{D/A} = 110\ \text{mm/s}^2\downarrow$

(a) We have $\qquad a_{C/B} = a_C - a_B = -60 \quad \text{or} \quad a_B = a_C + 60$

and $\qquad a_{D/A} = a_D - a_A = 110 \quad \text{or} \quad a_A = a_D - 110$

Substituting into Eqs. (2) and (4)

Eq. (2): $\qquad 2(a_D - 110) + 2(a_C + 60) + a_C = 0$

or $\qquad 3a_C + a_D = 100 \qquad (5)$

Eq. (4): $\qquad -(a_D - 110) - (a_C + 60) + 2a_D = 0$

or $\qquad -a_C + a_D = -50 \qquad (6)$

PROBLEM 11.59 (Continued)

Solving Eqs. (5) and (6) for a_C and a_D

$$a_C = 40 \text{ mm/s}^2$$

$$a_D = -10 \text{ mm/s}^2$$

Now $\qquad v_C = 0 + a_C t$

At $t = 3$ s: $\qquad v_C = (40 \text{ mm/s}^2)(3 \text{ s})$

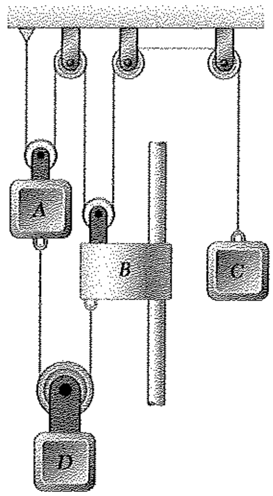
or $\qquad \mathbf{v}_C = 120 \text{ mm/s} \downarrow$ ◀

(*b*) We have $\qquad y_D = (y_D)_0 + (0)t + \frac{1}{2}a_D t^2$

At $t = 5$ s: $\qquad y_D - (y_D)_0 = \frac{1}{2}(-10 \text{ mm/s}^2)(5 \text{ s})^2$

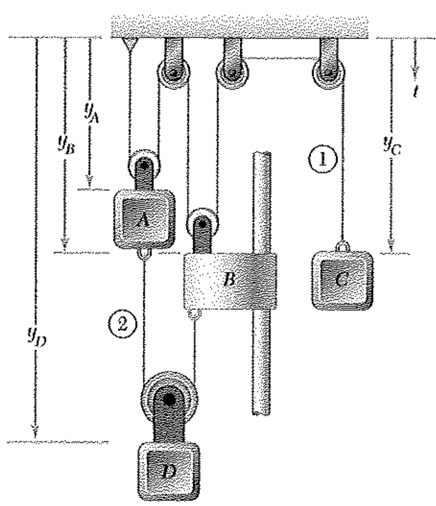
or $\qquad \mathbf{y}_D - (\mathbf{y}_D)_0 = 125 \text{ mm} \uparrow$ ◀

PROBLEM 11.60*



The system shown starts from rest, and the length of the upper cord is adjusted so that A, B, and C are initially at the same level. Each component moves with a constant acceleration, and after 2 s the relative change in position of block C with respect to block A is 280 mm upward. Knowing that when the relative velocity of collar B with respect to block A is 80 mm/s downward, the displacements of A and B are 160 mm downward and 320 mm downward, respectively, determine (*a*) the accelerations of A and B if $a_B > 10\ \text{mm/s}^2$, (*b*) the change in position of block D when the velocity of block C is 600 mm/s upward.

SOLUTION



From the diagram

Cable 1: $$2y_A + 2y_B + y_C = \text{constant}$$

Then $$2v_A + 2v_B + v_C = 0 \qquad (1)$$

and $$2a_A + 2a_B + a_C = 0 \qquad (2)$$

Cable 2: $$(y_D - y_A) + (y_D - y_B) = \text{constant}$$

Then $$-v_A - v_B - 2v_D = 0 \qquad (3)$$

and $$-a_A - a_B + 2a_D = 0 \qquad (4)$$

Given: At $$t = 0$$
$$v = 0$$
$$(y_A)_0 = (y_B)_0 = (y_C)_0$$

All accelerations constant at $t = 2$ s

$$y_{C/A} = 280\ \text{mm} \uparrow$$

When $$v_{B/A} = 80\ \text{mm/s} \downarrow$$
$$y_A - (y_A)_0 = 160\ \text{mm} \uparrow$$
$$y_B - (y_B)_0 = 320\ \text{mm} \downarrow$$
$$a_B > 10\ \text{mm/s}^2$$

PROBLEM 11.60* (Continued)

(*a*) We have $y_A = (y_A)_0 + (0)t + \frac{1}{2}a_A t^2$

and $y_C = (y_C)_0 + (0)t + \frac{1}{2}a_C t^2$

Then $y_{C/A} = y_C - y_A = \frac{1}{2}(a_C - a_A)t^2$

At $t = 2$ s, $y_{C/A} = -280$ mm:

$$-280 \text{ mm} = \frac{1}{2}(a_C - a_A)(2 \text{ s})^2$$

or $a_C = a_A - 140$ (5)

Substituting into Eq. (2)

$$2a_A + 2a_B + (a_A - 140) = 0$$

or $a_A = \frac{1}{3}(140 - 2a_B)$ (6)

Now $v_B = 0 + a_B t$

$$v_A = 0 + a_A t$$

$$v_{B/A} = v_B - v_A = (a_B - a_A)t$$

Also $y_B = (y_B)_0 + (0)t + \frac{1}{2}a_B t^2$

When $\mathbf{v}_{B/A} = 80 \text{ mm/s}\downarrow$: $80 = (a_B - a_A)t$ (7)

$$\Delta y_A = 160 \text{ mm}\downarrow: \quad 160 = \frac{1}{2}a_A t^2$$

$$\Delta y_B = 320 \text{ mm}\downarrow: \quad 320 = \frac{1}{2}a_B t^2$$

Then $160 = \frac{1}{2}(a_B - a_A)t^2$

Using Eq. (7) $320 = (80)t$ or $t = 4$ s

Then $160 = \frac{1}{2}a_A(4)^2$ or $\mathbf{a}_A = 20 \text{ mm/s}^2\downarrow$ ◀

and $320 = \frac{1}{2}a_B(4)^2$ or $\mathbf{a}_B = 40 \text{ mm/s}^2\downarrow$ ◀

Note that Eq. (6) is not used; thus, the problem is over-determined.

Alternative solution:

We have $$v_A^2 = (0) + 2a_A[y_A - (y_A)_0]$$

$$v_B^2 = (0) + 2a_B[y_B - (y_B)_0]$$

Then $$v_{B/A} = v_B - v_A = \sqrt{2a_B[y_B - (y_B)_0]} - \sqrt{2a_A[y_A - (y_A)_0]}$$

When $\mathbf{v}_{B/A} = 80 \text{ mm/s} \downarrow$:

$$80 \text{ mm/s} = \sqrt{2}\left[\sqrt{a_B(320 \text{ mm})} - \sqrt{a_A(160 \text{ mm})}\right]$$

or $$20 = \sqrt{2}\left(\sqrt{200_B} - \sqrt{100_A}\right) \qquad (8)$$

Solving Eqs. (6) and (8) yields $\mathbf{a}_A$ and $\mathbf{a}_B$.

(*b*) Substituting into Eq. (5)

$$a_C = 20 - 140 = -120 \text{ mm/s}^2$$

and into Eq. (4)

$$-(20 \text{ mm/s}^2) - (40 \text{ mm/s}^2) + 2a_B = 0$$

or $$a_D = 30 \text{ mm/s}^2$$

Now $$v_C = 0 + a_C t$$

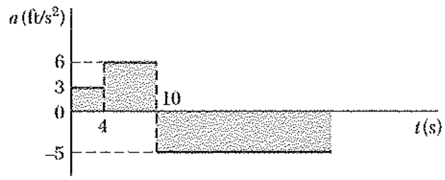
When $v_C = -600$ mm/s: $$-600 \text{ mm/s} = (-120 \text{ mm/s}^2)t$$

or $$t = 5 \text{ s}$$

Also $$y_D = (y_D)_0 + (0)t + \frac{1}{2}a_D t^2$$

At $t = 5$ s: $$y_D - (y_D)_0 = \frac{1}{2}(30 \text{ mm/s}^2)(5 \text{ s})^2$$

or $$\mathbf{y}_D - (\mathbf{y}_D)_0 = 375 \text{ mm} \downarrow \blacktriangleleft$$

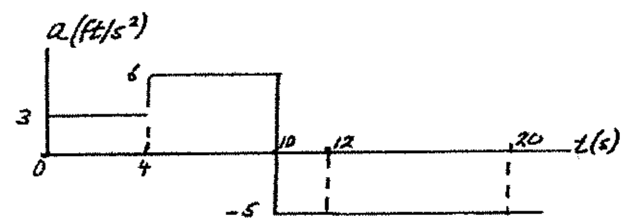


PROBLEM 11.61

A particle moves in a straight line with the acceleration shown in the figure. Knowing that it starts from the origin with $v_0 = -18$ ft/s, (*a*) plot the $v-t$ and $x-t$ curves for $0 < t < 20$ s, (*b*) determine its velocity, its position, and the total distance traveled when $t = 12$ s.

SOLUTION

(*a*)

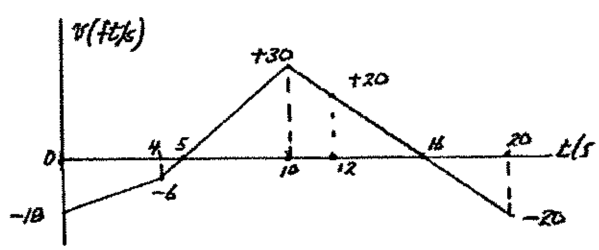


Initial conditions: $t = 0, \quad v_0 = -18 \text{ ft/s}, \quad x_0 = 0$

Change in v equals area under $a-t$ curve:

$$v_0 = -18 \text{ ft/s}$$

$0 < t < 4$ s:	$v_4 - v_0 = (3 \text{ ft/s}^2)(4 \text{ s}) = +12 \text{ ft/s}$	$v_4 = -6 \text{ ft/s}$
$4 \text{ s} < t < 10$ s:	$v_{10} - v_4 = (6 \text{ ft/s}^2)(6 \text{ s}) = +36 \text{ ft/s}$	$v_{10} = +30 \text{ ft/s}$
$10 \text{ s} < t < 12$ s:	$v_{12} - v_{10} = (-5 \text{ ft/s}^2)(2 \text{ s}) = -10 \text{ ft/s}$	$v_{12} = +20 \text{ ft/s}$ ◀
$12 \text{ s} < t < 20$ s:	$v_{20} - v_{12} = (-5 \text{ ft/s}^2)(8 \text{ s}) = -40 \text{ ft/s}$	$v_{20} = -20 \text{ ft/s}$



Change in x equals area under $v-t$ curve:

$$x_0 = 0$$

$0 < t < 4$ s:	$x_4 - x_0 = \frac{1}{2}(-18-6)(4) = -48 \text{ ft}$	$x_4 = -48 \text{ ft}$
$4 \text{ s} < t < 5$ s:	$x_5 - x_4 = \frac{1}{2}(-6)(1) = -3 \text{ ft}$	$x_5 = -51 \text{ ft}$